EDUMATSAINS
JURNAL PENDIDIKAN MATEMATIKA DAN SAINS
E-ISSN:2527-7235
Volume 11 Issue. 2, October 2026, pp 277-292
http://ejournal.uki.ac.id/index.php/edumatsains

From Context to Algebraic Procedures: A Praxeological Analysis of Knowledge Construction in a Gakko Tosho Mathematics Textbook

Ika Wulandari[1*], Didi Suryadi[2], Sufyani Prabawanto[3], Kusnandi[4]

[1,2,3,4]Pendidikan Matematika, FMIPA, Universitas Pendidikan Indonesia

e-mail: ikawulandari@upi.edu, didisuryadi@upi.edu, sufyani@upi.edu , kusnandi@upi.edu

Article Info
Article history:
Received: October 20th, 2026
Revised: October 24th, 2026
Accepted: October 26th, 2026
Available online: October 31st, 2026

https://doi.org/10.33541/edumatsains.v11i2.8822

Abstract

Mathematics textbooks not only introduce procedures for solving systems of linear equations in two variables but also provide mathematical reasons for their validity. This study examines the construction and justification of elimination and substitution techniques in the Indonesian Grade VIII Gakko Tosho student textbook (2021), using the teacher guide only to clarify and confirm task explanations. A qualitative document analysis was conducted within the Anthropological Theory of the Didactic. Seven types of tasks were examined through practical blocks (tasks and techniques) and theoretical blocks (technologies and theories), complemented by epistemic and systemic analyses. The findings reveal a progression from contextual problems and simultaneous solutions to elimination, substitution, strategy selection, and mathematical modeling. Elimination develops from relationships between contextual quantities before being formalized through operations on equations, whereas substitution draws on replacing expressions of equal value. The teacher guide explicitly provides operational justifications based on properties of equality, addition and subtraction of equations, and equal-valued substitution. Both resources also support checking solution pairs and comparing strategies. However, these explanations mainly concern particular operations and examples; a general account of why equivalent system transformations preserve the entire solution set was not found explicitly stated. The study identifies an established connection between contexts, techniques, and operational justifications while highlighting an opportunity to make the general mathematical basis of valid system transformations more visible in the organization of knowledge.

Keywords: praxeological analysis, algebraic techniques, mathematical justification, linear equation systems; mathematics textbooks

1. Introduction

Mathematics textbooks do more than present concepts and procedures; they also organize mathematical activities through which students can construct knowledge. Through problems, worked examples, explanations, and exercises, textbooks provide opportunities for students to understand the relationships among concepts, solution techniques, and their underlying mathematical justifications. Research indicates that the organization of textbook tasks shapes

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia

opportunities to learn, while textbook use is influenced by institutional contexts and teachers' decisions (Rezat et al., 2021; Rezat, 2024; Li & Wang, 2024; Ulusoy & İncikabı, 2020). Textbook studies also suggest that task types and cognitive demands may influence students' opportunities to develop conceptual understanding and mathematical reasoning (Berisha & Bytyqi, 2020; Polat & Dede, 2023; Takeuchi & Shinno, 2020). Therefore, textbook analysis should examine not only which techniques are introduced but also how they are constructed and mathematically justified.

In systems of linear equations in two variables, the relationship between techniques and justifications is particularly important because solving a system involves algebraic transformations that must preserve its solution set. Students need to understand that elimination and substitution are not merely sequences of symbolic operations but techniques grounded in properties of equality and relationships between equations. Such understanding enables students to explain why a transformation is valid rather than simply reproduce a demonstrated procedure. Previous research has identified persistent challenges in algebra learning concerning the meaning of symbols, translation between representations, algebraic operations, and mathematical reasoning (Castro et al., 2022; Grugeon-Allys & Pilet, 2024; Sibgatullin et al., 2022; Soniera, 2022). Studies of linear equation systems have also reported difficulties with mathematical connections, procedural work, and interpretation of solutions (Fatio et al., 2020; Hidayati et al., 2020; Pandiangan et al., 2022; Ario et al., 2025). These findings underscore the importance of examining how textbooks connect mathematical meaning, solution techniques, and the reasons that justify their use.

These relationships can be examined through the Anthropological Theory of the Didactic (ATD), which conceptualizes mathematical activity in terms of praxeological organizations. A praxeology comprises a practical block consisting of types of tasks (T) and techniques (τ), and a theoretical block comprising technologies (θ) and theories (Θ). A type of task specifies the mathematical work to be performed, while a technique describes how it is carried out. Technology provides explanations and justifications for a technique, whereas theory supplies a broader body of knowledge underpinning those explanations (Chevallard, 2006, 2019; Bosch & Gascón, 2014; Bosch et al., 2017). Within this framework, the relationship between praxis and logos allows researchers to investigate whether solution techniques are accompanied by adequate mathematical justifications. The perspective of didactic transposition also makes it possible to investigate how mathematical knowledge is organized into knowledge to be taught through textbooks (Bosch et al., 2021; Pansell, 2023; Gascón, 2024).

The closest studies clarify different aspects of textbook praxeology. Fardian et al. (2025) foreground systemic and epistemic dimensions of linear-equation tasks, whereas Agustito et al. (2025) examine the relations between techniques and their mathematical foundations in algebraic content. Marhami et al. (2024) analyze task-and-technique organizations for systems of two-variable linear equations in Indonesian textbooks. Rather than claiming the epistemic–systemic lens or the identification of task types as new, this study connects three analytical concerns in one task trajectory: how contextual comparisons generate elimination and substitution techniques, whether the available discourse justifies individual operations or preservation of the entire system solution set, and whether subsequent tasks reactivate those

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia

justifications. This provides a more specific account of how praxis and logos remain connected, change, or become less visible as techniques are formalized and practiced.

The Indonesian Grade VIII mathematics textbook by the Gakko Tosho team (2021) provides a relevant context for this investigation. Its chapter on systems of linear equations in two variables opens with a contextual problem involving two related quantities and develops the meaning of simultaneous solutions through ordered pairs that satisfy two equations. Elimination is introduced through a price-comparison situation before being expressed as algebraic operations, whereas substitution is presented after students have studied elimination. Students subsequently encounter varied forms of systems, strategy selection, and contextual problem solving. This organization offers an opportunity to trace how contextual meanings are transformed into algebraic techniques and how properties of equality are used to explain their validity. Thus, the present analysis examines not merely the order in which techniques appear but also the relationships among technique construction, mathematical justification, and continuity of knowledge.

This study therefore asks: (1) how tasks and techniques move from contextual situations to algebraic procedures; (2) how the technological and theoretical blocks justify those techniques, particularly with respect to system equivalence; and (3) how epistemic opportunities and systemic continuity support or limit these connections. Its theoretical contribution is an evidence-traceable distinction between operational justification, candidate verification, and solution-set equivalence within an ATD-based task trajectory, rather than another inventory of task types. Its practical implication is a criterion for textbook use and lesson planning: teachers can make reversible transformations and their mathematical reasons visible when moving from contextual elimination to routine procedures. These are document-based implications, not claims about actual student learning.

2. Methods

This qualitative document analysis applied the Anthropological Theory of the Didactic (ATD) to Chapter 2, “Systems of Linear Equations in Two Variables” (pp. 30–55), of the Indonesian Grade VIII student textbook entitled *Matematika untuk Sekolah Menengah Pertama Kelas VIII* (Tim Gakko Tosho, 2021b; adapted by M. Hafiizh and F. Y. Saptaningtyas; Ministry of Education, Culture, Research, and Technology). The companion *Buku Panduan Guru Matematika untuk Sekolah Menengah Pertama Kelas VIII* (Tim Gakko Tosho, 2021a; same ministry and year) was consulted only to clarify task directions, intended explanations, and worked solutions. The student textbook alone constituted the praxeological corpus; the teacher guide was not coded as an independent corpus.

The units were task sequences comprising activities, discussions, examples, and exercises in Chapter 2. Initial units were grouped by mathematical objective and the action required, not by the printed label or frequency of items. The resulting T1–T7 categories covered: a two-variable equation and its solutions (T1), simultaneous solutions (T2), contextual construction of elimination (T3), formal elimination (T4), substitution and strategy comparison (T5), varied algebraic forms (T6), and contextual modeling and checking (T7). Where one item served two purposes, its primary instructional demand determined its assigned category; secondary actions

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia

were recorded as techniques within that task type. End-of-section tests and three-variable enrichment were excluded. Chapter 3 links were treated only as cross-chapter context.

The analysis proceeded from repeated reading and page-level evidence logging to a T–τ–θ–Θ matrix: T denotes task type, τ the action or technique, θ the discourse justifying it, and Θ the broader mathematical principles reconstructed analytically (Chevallard, 2006; Bosch & Gascón, 2014). A justification was coded "explicit" if the student text stated the relevant rule or reason; "implicit" if it could be inferred from a displayed operation or question without being stated; and "not evident" if neither was identifiable. "Researcher-reconstructed" theory was always marked separately. Teacher-guide explanations could confirm a reading but never upgrade an implicit student-book justification to explicit. The analysis distinguished a valid operation, checking a candidate pair, and proving equivalence of entire solution sets.

For the supplemental epistemic assessment (Pritchard, 2018), perceptual required observable contexts, images, or tables; memorial required a prompt or task dependence on prior knowledge; introspective required explaining, comparing, or reviewing a technique; and a priori required a mathematical reason grounded in a definition, property, or logical relation. A pathway was "clearly afforded" (✓) when directly prompted or demonstrated, "limited/implicit" (△) when only indirectly available, and "not evident" (–) when no corresponding evidence appeared. Judgments describe documentary opportunities, not observed cognitive processes.

Systemic analysis traced prerequisites and transitions across T1–T7, including the shift from contextual difference to elimination and from algebraic techniques to modeling. A transition was regarded as coherent when the relevant concept or equality property had been made available before use; postponed links were noted without automatically classifying them as defects. Interpretive uncertainty was handled by rereading adjacent items, checking page-specific evidence against the student text, and consulting the teacher guide only for clarification. Decisions and exceptions were recorded in the analytical matrix. No independent second coder, consensus panel, expert validation, classroom observations, or student interviews were undertaken; thus, the study neither claims inter-rater agreement nor demonstrates actual learning obstacles (Fardian et al., 2025).

3. Result and Discussion

3.1. Knowledge-building task trajectory

Seven task groups were identified in the main sequence of Chapter 2. On pages 31–33, the textbook uses tables of solutions to connect quantities subject to two conditions. Pages 34–38 develop the subtraction of equal relations into the elimination technique. Pages 39–42 introduce substitution and transformations of algebraic forms, whereas pages 46–52 emphasize mathematical modeling and checking answers against the original situation. These task groups are analytical categories developed for this study rather than labels used by the textbook authors (Tim Gakko Tosho, 2021b).

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia

Table 1. Knowledge-building task types in the student textbook

Code	Type of task	Evidence in student textbook	Knowledge focus
T1	Form and interpret a linear equation in two variables (LETV) and its solutions	Chapter opening and Task 1, pp. 31–32	Relationships between two quantities, variables, LETV, solution pairs, and multiplicity of solutions
T2	Form a system and determine a simultaneous solution	Tasks 2–4, p. 33	Two conditions, two LETVs, and a pair satisfying both equations
T3	Develop the principle of elimination from a contextual situation	Mathematical activity, pp. 34–35	Subtracting two equal relations; equality properties; reduction to an equation in one variable
T4	Solve systems by elimination	Examples 1–3 and Tasks 1–5, pp. 36–38	Direct elimination, matching coefficients, back-substitution, and checking
T5	Solve systems by substitution and choose a strategy	Discussion, Example 4, Tasks 6–7, pp. 39–40	Equivalent expressions, comparison of elimination and substitution, and strategy choice
T6	Transform and solve systems in different forms	Examples 5–7 and Tasks 8–12, pp. 40–42	Parentheses, fractions, decimals, A=B=C, and equivalent algebraic reformulation
T7	Model, solve, and verify contextual problems using systems	Tasks 1–7, Examples 1–4 and “Consider”, pp. 46–52	Identifying relations and variables; modeling, solving, interpreting, and contextual checking

The classification reveals a progression from mathematical objects to transformations and applications. T1 and T2 provide experiences of identifying ordered pairs satisfying one or two relations. T3 provides a particular bridge between context and procedure: subtracting two combinations of prices becomes subtracting two equations. T4 and T5 draw upon equality properties to justify symbolic manipulation, while T6 broadens the range of algebraic forms. In T7, a numerical answer is not regarded as complete until its meaning has been checked against the initial situation.

3.2. Practical block: techniques extend beyond named methods

The practical block indicates that the textbook does not merely provide exercises in elimination and substitution. Techniques also include observing situations, completing tables, comparing sets of ordered pairs, applying equality properties, substituting equivalent expressions, and interpreting results. Thus, T2 and T4 involve different mathematical work although both eventually lead to a solution pair. This reading corresponds to the praxeological distinction between mathematical tasks and the techniques used to address them (Chevallard, 2019; Yunianta et al., 2023).

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia

Table 2. Practical-block summary for T1–T7.

Task	Technique (τ)	Operationalization in the student textbook
T1	τ1–τ4	Read quantitative relations from context or diagrams; define variables and express an LETV; calculate pairs and complete tables; identify and record ordered pairs satisfying the equation.
T2	τ1, τ3–τ6	Express a second condition as an LETV, complete the second table, compare both tables, and substitute candidate pairs into both equations.
T3	τ1, τ5–τ10	Compare two price situations, subtract corresponding quantities, express A=M and B=N, infer A−B=M−N, reduce to an equation in one variable, and substitute back.
T4	τ5, τ6, τ9, τ10	Add or subtract equations directly where possible; multiply equations to match coefficients when needed; solve the resulting one-variable equation; substitute back and check.
T5	τ10–τ13	Isolate a variable as an equivalent expression, replace it in the other equation, solve the resulting one-variable equation, and compare the appropriateness of elimination and substitution.
T6	τ9, τ13, τ14	Expand brackets, collect terms, clear fractions or decimals through permissible operations on both sides, interpret A=B=C as two equations, and choose a method.
T7	τ15–τ17; selected solving techniques	Identify relations from diagrams, tables, or text; define variables and formulate equations; select a technique; check against both equations and contextual restrictions; interpret the answer.

The contextual origin of elimination is particularly evident in T3 (student textbook, pp. 34–35). Its full coding is illustrated below; Table 2 maps the other task groups to their associated techniques.

Worked coding example (T3, student textbook, pp. 34–35). The task compares three hamburgers and one drink costing 750 with one hamburger and one drink costing 350. T = construct elimination from a contextual difference; τ = set $3x + y = 750$ and $x + y = 350$, subtract to obtain $2x = 400$, then determine $x = 200$ and check it against the original relations. θ = the displayed equalities permit subtraction of corresponding sides ($A = M$ and $B = N$ imply $A - B = M - N$): this is an operational justification. Θ = equality properties and reversible transformations of linear systems (researcher reconstruction). The subtraction principle is explicit, whereas preservation of the complete solution set is not stated as a general theorem in this episode. The teacher guide confirms the equality-based teaching intention but does not create another task code.

On pages 39–40, substitution is introduced by replacing a variable with an equal-valued expression. Students are subsequently asked to compare elimination and substitution and select a suitable method. Hence the criteria for choosing a technique begin to depend on equation structure. Justifying this choice is distinct from merely completing the calculation (Ayala-Altamirano & Molina, 2021; Gök & Erdoğan, 2023).

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia

3.3. Theoretical block: explicit and implicit mathematical justifications

The reasons for techniques differ in how explicitly they appear in the student textbook. A solution is directly defined as a pair making an equation true on pages 32–33. Equality properties supporting addition and subtraction are presented on pages 35–36, and substitution is explained as replacement by an equal-valued expression on page 39. However, "equivalent systems" and a general statement that permissible system transformations preserve the complete solution set are not explicitly developed as objects of study in the analyzed pages.

Table 3. Reconstructed theoretical block; theory labels are researcher-assigned, not necessarily explicit in the student textbook

Task	Technology (θ)	Theory (Θ): analytical reconstruction	Interpretive note
T1	θ1: an ordered pair is a solution if it makes an LETV true; one LETV may have multiple solutions.	Θ1: variables, linear equations, and equality properties in school algebra.	The teacher's guide makes the extension to the real-number domain clearer than the student book.
T2	θ2: a solution to a system makes both equations true simultaneously.	Θ2: linear systems and simultaneous satisfaction of two relations.	The meaning of solution is built through comparison of tables and checking.
T3	θ3: adding/subtracting equal relations produces a valid equation and may eliminate a variable.	Θ1/Θ3: equality properties and algebraic transformations of linear equations.	The text uses A=M, B=N, and A−B=M−N; invariance of the full solution set is not formalized.
T4	θ4: coefficients may be matched by scaling equations, then a variable eliminated by addition/subtraction.	Θ3: equality properties and transformations of linear equations.	Checking establishes whether a candidate satisfies both original equations.
T5	θ5: an expression equal to a variable may replace it elsewhere; suitability depends on system structure.	Θ1/Θ3: substitution and equality of expressions.	Comparing methods supports justification of strategy choice.
T6	θ6: nonstandard equations may be converted to solvable forms using permissible algebraic steps.	Θ1/Θ3: distributivity, operations on both sides, and linear-equation structure.	System equivalence across transformations is mainly implicit.

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia

Task	Technology (θ)	Theory (Θ): analytical reconstruction	Interpretive note
T7	θ7: a model must express relevant quantitative relations; its solution must fit the equations and context.	Θ2/Θ4: linear systems and mathematical modeling.	Contextual verification is an explicit part of solving.

It is essential to distinguish a justification actually stated in the student textbook from a broader theory reconstructed by the researcher. For example, multiplying an equation by a non-zero number and adding or subtracting equations are grounded in equality properties. Yet a general proposition specifying when two systems have exactly the same solution set can justify these operations mathematically without necessarily having been institutionalized for students. In ATD, the relation between technique and technology must therefore be analyzed through the justifying discourse available, not inferred solely from the correctness of a computed answer (Bosch & Gascón, 2014; Florensa et al., 2020; Rojas Suárez & Sierra Delgado, 2022).

The teacher's guide supports this distinction by elaborating the intended equality-based explanations of elimination and substitution. This supporting explanation is reported as confirmation of the interpretation of student-book tasks, not as evidence that every general principle is printed explicitly for students. In particular, verifying one computed pair against the original equations is not logically identical to demonstrating that an entire solution set is preserved by a reversible transformation (Tim Gakko Tosho, 2021a, 2021b).

3.4. Local praxeological organizations and their relationships

Three local praxeological organizations can be reconstructed from the main chapter. pG1 establishes the meaning of pairs satisfying two equations, pG2 develops algebraic transformations for determining these pairs, and pG3 returns the solution to quantitative relationships. The three are interdependent: pG2 requires the notion of solution built in pG1, and pG3 draws upon both the mathematical model and the available solving techniques.

Table 4. Local organizations and the cross-chapter extension (*not included in the core praxeological corpus)

Organization	Focus	Praxis	Logos
pG1	Meaning of LETVs, systems, and simultaneous solutions	T1–T2 / τ1–τ6	θ1–θ2 / principally Θ1–Θ2
pG2	Algebraic transformation and solving	T3–T6 / τ5–τ14	θ3–θ6 / principally Θ1–Θ3
pG3	Modeling, application, and verification	T7 / τ15–τ17 and selected solving techniques	θ7 / principally Θ2 and Θ4

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia

Organization	Focus	Praxis	Logos
pG4*	Graphical representation and number of solutions (cross-chapter reference)	Chapter 3 tasks, not counted in Chapter 2	Intersection as solution; parallel/coincident lines; coordinate geometry and linear functions

Graphical representations and the classification of solution multiplicity are treated in the supporting analytical materials as a continuation in Chapter 3. They are not counted as task types in the Chapter 2 corpus. This cross-chapter link should be described as contextual information about the textbook's organization rather than as an additional local praxeology analyzed to the same evidential depth. Where Chapter 3 is not directly examined, its specific page-level details require separate verification.

3.5. Epistemic opportunities afforded by the tasks

Four epistemic pathways were used as additional analytical lenses for opportunities to construct and justify knowledge. Perceptual opportunities involve examining contexts, drawings, or tables; memorial opportunities involve recalling prior knowledge of equations; introspective opportunities involve explaining, comparing, or reviewing one's approach; and a priori opportunities involve reasoning from equality properties and logical relationships. These are indicators of possibilities afforded by documents, not evidence that students actually engaged in all four processes (Pritchard, 2018; Fardian et al., 2025).

Table 5. Epistemic opportunities; ✓ clearly afforded, △ limited or partly implicit

Task	Perceptual	Memorial	Introspective	A priori	Assessment	Brief evidence
T1	✓	✓	✓	△	Partial	Context, tables and discussion are prominent; generalization to a formal LETV is largely supplied by the text.
T2	✓	✓	✓	✓	Relatively complete	Comparison of two tables, prior LETV knowledge, common pairs, and verification.
T3	✓	✓	✓	✓	Relatively complete	Price diagrams, prior equality properties, comparison of solutions, and A−B=M−N.
T4	✓	✓	△	✓	Partial	Equality-based reasoning is available, but many exercises focus on carrying out the algorithm.
T5	✓	✓	✓	✓	Relatively complete	Attention to algebraic forms, strategic comparison, and equivalent-expression substitution.

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia

Task	Perceptual	Memorial	Introspective	A priori	Assessment	Brief evidence
T6	✓	✓	✓	✓	Relatively complete	Structural recognition, distributivity, both-side operations, and logical transformations.
T7	✓	✓	✓	✓	Relatively complete	Contextual representations, modeling, prior system knowledge, and checking results.

T2, T3, and T5 show relatively clear links among all four pathways. Students compare data, activate earlier knowledge, review techniques, and apply mathematical reasons. In T4 the textbook provides a foundation for elimination, but many subsequent exercises emphasize obtaining answers rather than explicitly reflecting on every step. This does not mean that elimination lacks meaning; rather, the issue is whether the reasons established during its introduction are regularly reactivated during practice. This distinction relates to concerns about the balance of procedural fluency and conceptual understanding in algebra materials (Agustito et al., 2025; Henriques & Martins, 2022).

3.6. Systemic coherence and potential limitations

The T1–T7 sequence is generally progressive. One linear relation is explored before simultaneous relations; subtraction of equal relations precedes more complex elimination; substitution follows the introduction of system solutions; and established techniques are subsequently used in contextual modeling. Therefore, presenting elimination before substitution is not intrinsically non-systemic simply because another textbook or instructional design uses the reverse order. Systemic judgment depends on the availability of prerequisites and connections between tasks.

Table 6. Knowledge transitions and conditional didactic risks

Transition	Prerequisite or knowledge connection	Assessment	Potential risk if disconnected
Prior one-variable equations → T1	Grade VII variables, one-variable linear equations and operations on both sides.	Coherent	Equality may be treated as arbitrary symbol movement rather than a preserved relation.
T1 → T2	LETV and its solution pairs underpin two simultaneous conditions.	Coherent	Separate solutions may be confused with pairs satisfying both equations.
T2 → T3	System meaning precedes reduction using equality properties.	Coherent	Skipping the contextual principle could make elimination seem purely mechanical.
T3 → T4	Subtraction/addition principles precede formal elimination routines.	Coherent	The reasons for maintaining valid solutions may fade during practice.

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia

Transition	Prerequisite or knowledge connection	Assessment	Potential risk if disconnected
T4 → T5	Reduction to a one-variable equation prepares an alternative reduction by substitution.	Coherent	Substitution might become an unexplained replacement rule.
T5 → T6	Both techniques are available before structurally more complex systems.	Coherent	Invalid transformations or errors with signs and fractions become possible.
T6 → T7	Concepts and methods are available before varied modeling situations.	Coherent	If solving dominates, identifying relationships and contextual meaning may be neglected.
Chapter 2 → Chapter 3	The geometric meaning and multiplicity of solutions follow work on linear functions and graphs.	Cross-chapter connection; not complete in Chapter 2	Students might assume all systems have a unique pair or isolate graphs from solution sets.

Two potential vulnerabilities warrant closer examination. First, the general justification for equivalent systems is not explicitly formulated as a unifying principle across the transformations. Students who encounter procedures without retaining the underlying reasons could separate symbol manipulation from the meaning of a solution. Second, the geometric interpretation and three possible numbers of solutions are not developed as a major task sequence within the Chapter 2 corpus. This is not necessarily a weakness of the book as a whole because conceptual organization can extend across chapters; the link must nevertheless be activated for students to avoid treating every system as if it had exactly one solution. These are didactic hypotheses, not findings of observed student learning obstacles (Fardian et al., 2025; Risfiandina & Sumiaty, 2026).

For modeling, the student textbook explicitly describes identifying quantitative relationships, forming equations, solving, and checking whether answers are reasonable in context (p. 47). Tasks involving food prices, museum tickets, travel and changing quantities offer varied interpretations of simultaneous relations. Contextual verification is a strength because it links calculation with interpretation; work on algebraic representation helps explain its importance (Soniera, 2022; Castro et al., 2022; Hidayati et al., 2020). Actual classroom enactment remains beyond the scope of this document analysis.

The literature on algebra errors calls attention to symbol meaning and transitions between operations; it provides a reason for caution when interpreting procedural practice, not evidence that students using this book made those errors (Pramesti & Retnawati, 2019). More broadly, the findings underline why praxeology should be read as an organization. Listing techniques without tracing their technology reduces evaluation to procedural inventories, while evaluating isolated examples without surrounding tasks risks disproportionate criticism (Artigue & Winsløw, 2010; Florensa et al., 2020; Rezat, 2024).

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia

4. Conclusion

The analysis of Chapter 2 of the Gakko Tosho student textbook identified seven task groups organized into three local praxeologies: interpreting LETVs and simultaneous solutions (pG1), constructing algebraic techniques through elimination, substitution and varied equation forms (pG2), and modeling and verifying contextual solutions (pG3). The sequence is generally systemically coherent because key concepts and prerequisites are introduced before they are required in more demanding tasks.

A major strength is the progression from contextual differences between quantities to equality-based elimination, the introduction of substitution through equivalent expressions, discussion of strategy choice, and verification within real-world contexts. Accordingly, the student textbook should not be characterized as presenting algorithms without justification. Explanations in the teacher's guide confirm the intended instructions and worked solutions but are not analyzed as a separate praxeological corpus.

Although operational justifications are available, a general formulation of system equivalence and the preservation of the entire solution set is not made explicit within the analyzed chapter. Geometrical meaning and the possible numbers of solutions are likewise located outside its main task sequence, with relevant connections developed elsewhere in the textbook. These features indicate differences in the visibility and placement of knowledge, not empirically established obstacles experienced by students. The contribution is to clarify how contextual technique construction, mathematical justifying discourse and task continuity are connected. Testing their didactic consequences would require classroom evidence and student responses.

5. Aknowledgments

Thanks to Indonesian Education Scholarship (*Beasiswa Pendidikan Indonesia*/BPI) and the Center for Higher Education Financing and Assessment (*Pusat Pembiayaan dan Asesmen Pendidikan Tinggi*/ PPAPT) under the Ministry of Education, Culture, Research, and Technology; and the Indonesia Endowment Fund for Education (*Lembaga Pengelola Dana Pendidikan*/ LPDP) under the Ministry of Finance of the Republic of Indonesia.

6. References

Agustito, D., Kuncoro, K. S., Kusumaningrum, B., Trisniawati, Sukiyanto, & Wijayanti, D. (2025). Praxeological analysis of linear algebra content presentation: A case study of Indonesian mathematics textbooks. *EURASIA Journal of Mathematics, Science and Technology Education*, 21(6), em2648. https://doi.org/10.29333/ejmste/16508

Ario, M., Suhendra, Jupri, A., & Nurlaelah, E. (2025). Students' errors and learning obstacles in solving algebraic word problems: Hermeneutic phenomenology. *Education Sciences*, 15(12), 1674. https://doi.org/10.3390/educsci15121674

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia

Artigue, M., & Winsløw, C. (2010). International comparative studies on mathematics education: A viewpoint from the anthropological theory of didactics. *Recherches en Didactique des Mathématiques*, 30(1), 47–82.

Ayala-Altamirano, C., & Molina, M. (2021). Fourth-graders' justifications in early algebra tasks involving a functional relationship. *Educational Studies in Mathematics*, 107(2), 359–382. https://doi.org/10.1007/s10649-021-10036-1

Berisha, V., & Bytyqi, R. (2020). Types of mathematical tasks used in secondary classroom instruction. *International Journal of Evaluation and Research in Educatio*n, 9(3), 751–758. https://doi.org/10.11591/ijere.v9i3.20617

Bosch, M., & Gascón, J. (2014). Introduction to the Anthropological Theory of the Didactic (ATD). In A. Bikner-Ahsbahs & S. Prediger (Eds.), *Networking of theories as a research practice in mathematics education* (pp. 67–83). Springer. https://doi.org/10.1007/978-3-319-05389-9_5

Bosch, M., Gascón, J., & Trigueros, M. (2017). Dialogue between theories interpreted as research praxeologies: The case of APOS and the ATD. *Educational Studies in Mathematics*, 95(1), 39–52. https://doi.org/10.1007/s10649-016-9734-3

Bosch, M., Hausberger, T., Hochmuth, R., Kondratieva, M., & Winsløw, C. (2021). External didactic transposition in undergraduate mathematics. *International Journal of Research in Undergraduate Mathematics Education*, 7(1), 140–162. https://doi.org/10.1007/s40753-020-00132-7

Castro, E., Cañadas, M. C., Molina, M., & Rodríguez-Domingo, S. (2022). Difficulties in semantically congruent translation of verbally and symbolically represented algebraic statements. *Educational Studies in Mathematics*, 109(3), 593–609. https://doi.org/10.1007/s10649-021-10088-3

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. *Proceedings of the 4th Congress of the European Society for Research in Mathematics Education* (pp. 21–30).

Chevallard, Y. (2019). Introducing the anthropological theory of the didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education*, 12, 71–114. https://doi.org/10.24529/hjme.1205

Fardian, D., Suryadi, D., & Prabawanto, S. (2024). Students' learning obstacles in solving mathematical proficiency tasks: A hermeneutic phenomenological study focused on algebra. *Kreano, Jurnal Matematika Kreatif-Inovatif*, 15(2), 593–618. https://doi.org/10.15294/36aygs91

Fardian, D., Suryadi, D., & Prabawanto, S. (2025). A praxeological analysis of linear equations in Indonesian mathematics textbooks: Focusing on systemic and epistemic aspect. *Journal on Mathematics Education*, 16(1), 225–254. https://doi.org/10.22342/jme.v16i1.pp225-254

Fatio, N. A., Fatimah, S., & Rosjanuardi, R. (2020). The analysis of students' learning difficulties on system of linear equation in two variables topic. *Journal of Physics: Conference Series*, 1521, 032062. https://doi.org/10.1088/1742-6596/1521/3/032062

Florensa, I., Bosch, M., & Gascón, J. (2020). Reference epistemological model: What form and function in school institutions? *Educação Matemática Pesquisa*, 22(4), 240–249. https://doi.org/10.23925/1983-3156.2020v22i4p240-249

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia

Gascón, J. (2024). Contributions of the anthropological theory of the didactic to the epistemological programme of research in mathematics education. *ZDM–Mathematics Education*, 56, 1319–1330. https://doi.org/10.1007/s11858-024-01563-1

Gök, M., & Erdoğan, A. (2023). Didactic praxeologies employed by mathematics teachers in teaching the inverse function. *Journal of Computer and Education Research*, 11(22), 1089–1112. https://doi.org/10.18009/jcer.1361502

Grugeon-Allys, B., & Pilet, J. (2024). The algebraic learning of middle school students' evolution during a school year: A statistical large-scale study based on results in mathematics didactics. *Educational Studies in Mathematics*, 117, 533–564. https://doi.org/10.1007/s10649-024-10347-z

Hendriyanto, A., Suryadi, D., Dahlan, J. A., & Juandi, D. (2023). Praxeology review: Comparing Singaporean and Indonesian textbooks in introducing the concept of sets. *EURASIA Journal of Mathematics, Science and Technology Education*, 19(2), em2229. https://doi.org/10.29333/ejmste/12953

Henriques, A., & Martins, M. A. F. (2022). Mathematical reasoning in linear systems learning: A higher education exploratory teaching experiment with prospective teachers. *Avances de Investigación en Educación Matemática*, 21, 65–85. https://doi.org/10.35763/aiem21.4238

Hidayati, V. R., Maulyda, M. A., Gunawan, G., Rahmatih, A. N., & Erfan, M. (2020). System of linear equation problem solving: Descriptive-study about students' mathematical connection ability. *Journal of Physics: Conference Series*, 1594, 012042. https://doi.org/10.1088/1742-6596/1594/1/012042

Kuncoro, K. S., Suryadi, D., Dahlan, J. A., & Jupri, A. (2024). Praxeological analysis in Indonesian and Singaporean mathematics textbooks: An understanding geometrical similarity by students. *Journal on Mathematics Education*, 15(4), 1197–1218. https://doi.org/10.22342/jme.v15i4.pp1197-1218

Li, F., & Wang, L. (2024). A study on textbook use and its effects on students' academic performance. *Disciplinary and Interdisciplinary Science Education Research*, 6(1), 4. https://doi.org/10.1186/s43031-023-00094-1

Marhami, M., Suryadi, D., & Dasari, D. (2024). System of linear equations in two variables: A praxeological analysis of Indonesian textbooks. In A. A. Khan, M. Demirbilek, & M. L. Ciddi (Eds.), *Proceedings of ICSEST 2024—International Conference on Studies in Engineering, Science, and Technology* (pp. 112–124). ISTES.

Pandiangan, Y., Santosa, C. A. H. F., & Novaliyosi, N. (2022). Analisis kesalahan siswa dalam menyelesaikan masalah matematis pada materi SPLDV berdasarkan Newman's Error Analysis. *Jurnal Amal Pendidikan*, 3(1), 35–52. https://doi.org/10.36709/japend.v3i1.24004

Pansell, A. (2023). Mathematical knowledge for teaching as a didactic praxeology. *Frontiers in Education*, 8, 1165977. https://doi.org/10.3389/feduc.2023.1165977

Polat, S., & Dede, Y. (2023). Trends in cognitive demands levels of mathematical tasks in Turkish middle school mathematics textbooks: Algebra learning domain. *International Journal for Mathematics Teaching and Learning*, 24(1), 40–61. https://doi.org/10.4256/ijmtl.v24i1.476

Pramesti, T. I., & Retnawati, H. (2019). Difficulties in learning algebra: An analysis of students' errors. *Journal of Physics: Conference Series*, 1320, 012061. https://doi.org/10.1088/1742-6596/1320/1/012061

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia

Pritchard, D. (2018). What is this thing called knowledge? (4th ed.). Routledge. https://doi.org/10.4324/9781351980326

Pulungan, R. R., & Suhendra. (2019). Analysis of student's misconception in solving system of linear equation in two variables. *Journal of Physics: Conference Series*, 1157(4), 042113. https://doi.org/10.1088/1742-6596/1157/4/042113

Rezat, S. (2024). Research on curriculum resources in mathematics education: A survey of the field. *ZDM–Mathematics Education*, 56(2), 223–237. https://doi.org/10.1007/s11858-024-01559-x

Rezat, S., Fan, L., & Pepin, B. (2021). Mathematics textbooks and curriculum resources as instruments for change. *ZDM–Mathematics Education*, 53(6), 1189–1206. https://doi.org/10.1007/s11858-021-01309-3

Risfiandina, R., & Sumiaty, E. (2026). Analysis of students' learning obstacles in solving SLETV problems and the development of a hypothetical didactical design based on contextual problems. *Journal on Mathematics Education Research*, 7(1), 160–174. https://doi.org/10.17509/j-mer.v7i1.930

Rojas Suárez, C., & Sierra Delgado, T. Á. (2022). A reference epistemological model regarding the determination and construction of solids for compulsory secondary education. *Acta Scientiae*, 24(8), 437–475. https://doi.org/10.17648/acta.scientiae.7185

Sibgatullin, I. R., Korzhuev, A. V., Khairullina, E. R., Sadykova, A. R., Baturina, R. V., & Chauzova, V. (2022). A systematic review on algebraic thinking in education. *EURASIA Journal of Mathematics, Science and Technology Education*, 18(1), em2065. https://doi.org/10.29333/ejmste/11486

Soniera, C. (2022). The use of representations when solving algebraic word problems and the sources of solution errors. *International Journal of Science and Mathematics Education*, 20(5), 1037–1056. https://doi.org/10.1007/s10763-021-10181-2

Takeuchi, H., & Shinno, Y. (2020). Comparing the lower secondary textbooks of Japan and England: A praxeological analysis of symmetry and transformations in geometry. *International Journal of Science and Mathematics Education*, 18(4), 791–810. https://doi.org/10.1007/s10763-019-09982-3

Ulusoy, F., & İncikabı, L. (2020). Middle school teachers' use of compulsory textbooks in instruction of mathematics. *International Journal for Mathematics Teaching and Learning*, 21(1), 1–18. https://doi.org/10.4256/ijmtl.v21i1.227

Utami, N. S., Prabawanto, S., & Priatna, N. (2022). A didactical design for introducing the concepts in algebraic forms using the theory of praxeology. *Beta: Jurnal Tadris Matematika*, 15(1), 53–71. https://doi.org/10.20414/betajtm.v15i1.508

Utami, N. S., Prabawanto, S., & Suryadi, D. (2024). How do Indonesian students learn function concepts? A praxeological analysis of textbook. *Journal on Mathematics Education*, 15(2), 451–472. https://doi.org/10.22342/jme.v15i2.pp451-472

Yunianta, T. N. H., Suryadi, D., Dasari, D., & Herman, T. (2023). Textbook praxeological-didactical analysis: Lessons learned from the Indonesian mathematics textbook. *Journal on Mathematics Education*, 14(3), 503–524. https://doi.org/10.22342/jme.v14i3.pp503-524

Tim Gakko Tosho. (2021a). *Buku panduan guru matematika untuk sekolah menengah pertama kelas VIII*. Kementerian Pendidikan, Kebudayaan, Riset, dan Teknologi.

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia

Tim Gakko Tosho. (2021b). *Matematika untuk sekolah menengah pertama kelas VIII* (M. Hafiizh & F. Y. Saptaningtyas, adapters). Kementerian Pendidikan, Kebudayaan, Riset, dan Teknologi.

This is an open access article under the HYPERLINK "https://creativecommons.org/licenses/by/4.0/" *CC BY 4.0 license.*
Copyright ©2025 by Author. Published by Universitas Kristen Indonesia