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Learning Obstacles and Students' Statistical Thinking Levels in Elementary Schools: A Foundation for Designing a Hypothetical Learning Trajectory

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Abstract

Statistical thinking is an essential 21st-century competency; however, statistics instruction in elementary schools remains largely procedural. This study integrates learning obstacles, statistical thinking levels, and Hypothetical Learning Trajectory (HLT) within a unified analytical framework. It aims to identify students' learning obstacles and statistical thinking levels in learning measures of central tendency as an empirical basis for HLT design. A qualitative case study was conducted with 16 sixth-grade students and one teacher at SDN 1 Waringinsari in June 2026. Data were collected through diagnostic tests, participatory observation, and semi-structured interviews and analyzed using NVivo 14 through open, axial, and selective coding. The findings indicate that epistemological obstacles predominate, followed by ontogenical and didactical obstacles. Students' statistical thinking levels are concentrated at transitional and quantitative levels, while the analytical level is rarely achieved. Epistemological obstacles persist across transitional and quantitative levels. The study contributes a systematic mapping of learning obstacles and statistical thinking levels to inform evidence-based HLT design.

Keywords: *Learning obstacles*; *statistical thinking*; *measures of central tendency*; *hypothetical learning trajectory*; *elementary school*

1. Introduction

In the era of data-driven learning, statistical thinking is an essential 21st-century competency in mathematics education, as it enables students to collect, organize, analyze, and interpret data to support informed decision-making. Nevertheless, according to the results of Programme for

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International Student Assessment (PISA) in 2022 which were published in 2023, mathematical literacy of the Indonesian students still falls below the OECD average, especially in reasoning and data interpretation (OECD, 2023). Such situation shows that there is a need to change approaches in the process of mathematics teaching by focusing not only on procedures, but also on concepts. Such conclusion can be drawn from the research done in Indonesia and showing that primary school students have some problems with the measures of central tendency. For example, in their research, Rahayu et al. (2024) stated that students had problems with the concepts of mean, median, and mode and using them in the process of working with different representations of data. In its turn, Wirayuda et al. (2025) stated that students could use the formulas, but did not understand them and made mistakes while choosing the most appropriate measures of central tendency. However, Sari et al. (2025) showed that the Indonesian Realistic Mathematics Education (PMRI) approach to the process of instruction can help in developing conceptual understanding.

Based on these studies, previous research has mostly focused on either the effectiveness of instructional models or the development of learning materials. Research that has specifically identified the learning obstacles and statistical thinking levels of students as the basis for constructing Hypothetical Learning Trajectory (HLT) of measures of central tendency at the elementary school level is still scarce. Hence, the purpose of this study is to identify the learning obstacles and statistical thinking levels of students empirically in order to develop an HLT based on the characteristics of learning at elementary school. Procedural teaching becomes quite ineffective especially when it comes to measures of central tendency. Although the students can calculate mean, median, and mode accurately, they usually have difficulties in choosing which measure suits the characteristics of the data distribution. Such obstacles can be due to the textbooks that offer insufficient chances for different types of reasoning and the way of teaching that introduces formula without following a process of knowledge construction (Parra-Fica et al., 2024). The fundamental problems in the interpretation and comprehension of data representations will later become part of the systematic problems of cognition for the learners at advanced grades (Parra-Fica et al., 2024).

Different researches suggest that problems in learning of basic statistics in schools result from the complicated interaction between learners' cognitive readiness, poor instruction, and lack of conceptual knowledge of mathematics (Melati et al., 2024). The creation of HLT is one of the ways to overcome the problems mentioned above because it allows designing an effective learning trajectory from informal knowledge to formal mathematical understanding (Clements & Sarama, 2011). An HLT grounded in realistic learning contexts can provide a systematic progression from real-world situations to formal concepts, enabling students to construct conceptual understanding and solve problems involving measures of central tendency. Such learning processes can support the development of students' statistical thinking (Utari et al., 2025).

Nevertheless, the existing literature tends to examine learning obstacles, statistical thinking levels, and HLT separately. Previous studies have primarily focused on identifying and classifying learning obstacles in mathematics education (Brousseau & Balacheff, 2002; Suryadi, 2013), examining the development of students' statistical thinking based on specific thinking levels without directly connecting these levels to instructional design (Jones et al., 2000), or developing

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HLTs without first conducting a comprehensive empirical analysis of students' learning obstacles and statistical thinking characteristics (Clements & Sarama, 2011; Gravemeijer, 2004). Therefore, studies combining learning barriers and levels of statistical thinking as an empirical foundation for HLT development are rare. The importance of conducting a holistic preliminary study becomes a necessity in order to guarantee that the proposed learning trajectory aligns with the learners' needs and is prepared for any potential conceptual obstacles (Suryadi, 2013).

Accordingly, this study will be concentrated on providing a thorough investigation of students' learning obstacles and statistical thinking levels in measures of central tendency. Separation of learning obstacles and statistical thinking levels from the HLT design could result in inadequate consideration of specific difficulties that students face or variations in their statistical thinking. As a result, instructional activities could be rather general and would not offer adequate assistance to students with various levels of knowledge. Integration of the mentioned components is necessary to make sure that instructional choices are based on the difficulties that students have and the level of their statistical thinking. The uniqueness of this study is connected with the connection between learning obstacles identification and statistical thinking levels and the design of an adaptive Local Learning Trajectory for elementary school students. In particular, this study will aim to: (1) identify learning obstacles of students regarding mean, median and mode; (2) classify the level of students' statistical thinking and (3) provide instructional recommendations to create an adaptive Local Learning Trajectory for elementary school students.

2. Methods

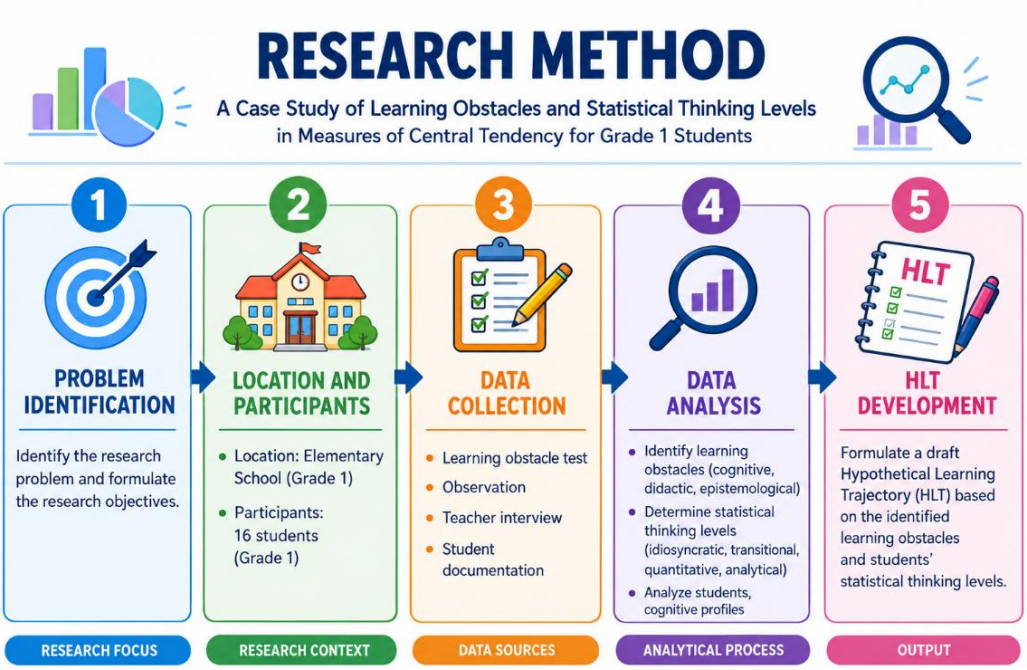
This study employed a qualitative case study design to investigate students' learning obstacles and statistical thinking levels in learning measures of central tendency. The study was conducted at SDN 1 Waringinsari and involved 16 sixth-grade students selected through purposive sampling. Data were collected through learning obstacle diagnostic tests, classroom observations, interviews with students and the teacher, and documentation of students' written work. The data were analyzed qualitatively to identify learning obstacles categorized as ontogenetic, didactical, and epistemological, and to map students' statistical thinking levels into four categories: idiosyncratic, transitional, quantitative, and analytical. The findings were subsequently used as an empirical basis for developing a Hypothetical Learning Trajectory (HLT) aligned with students' learning obstacles and developmental patterns of statistical thinking.

The number of participants was considered adequate because this study adopted a qualitative case study approach emphasizing in-depth analysis of information-rich cases through data triangulation. Triangulation involved diagnostic tests, classroom observations, semi-structured interviews, and analysis of students' written work. During the analysis, recurring patterns of learning obstacles and statistical thinking levels were identified, with no substantially new categories emerging. The data were therefore considered to have reached saturation. Consequently, the study was not concerned with statistical representation, but with giving an in-depth description

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of the problems encountered by students in their studies and levels of statistical thinking as the basis for formulating an adaptive Hypothetical Learning Trajectory (HLT).

Figur 1.
Research Method



Data Collection Instruments

The data collection instruments were designed in three forms: (1) Learning Obstacle Test, consisting of an open-ended instrument containing five essay questions designed to identify students' difficulties in statistical thinking. The questions assessed students' ability to analyze the appropriateness of measures of central tendency (Question 1), understand the concept of the mean (Question 2), determine and interpret the median (Question 3), determine and interpret the mode (Question 4), and organize and represent data appropriately (Question 5); (2) Participatory Observation, involving direct observation to document students' interaction patterns, hesitation, difficulties, and spontaneous strategies while completing the test; and (3) Semi-Structured Interviews, conducted with selected key informants to obtain verbal clarification and explore the reasoning underlying their written responses.

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Instrument Validity and Reliability

The research tools were made up of the test for learning obstacles, semi-structured interview guidelines, learning observation sheets, and Hypothetical Learning Trajectory (HLT). Before conducting the study, all the research tools were subject to validity by judgment of three lecturers in Mathematics Education that specialize in the learning of mathematics and statistics education. The validity process entailed judging the alignment of each tool with the research objectives, statistical thinking indicators, appropriateness of language, relevance of the problem contexts, and measurability of the intended aspects. After receiving the feedback and suggestions from the validators, all research tools were revised accordingly till all of them were deemed valid and appropriate.

Trustworthiness of the research has been ensured by using the same set of guidelines for the instrument, systematic application of the data collection process, and methodological triangulation with the learning obstacle test, learning observation, semi-structured interview, and analysis of the students' writing. Moreover, the process of data analysis has been carried out systematically by going through the steps of data reduction, data display, and conclusion drawing, which has been based on the theory of learning obstacles and statistical thinking levels. Methodological triangulation and systematic documentation of the process of analysis have been used to increase consistency and credibility of the research results.

Data Analysis

Data analysis used the interactive coding approach proposed by Miles et al. (2018) and was conducted with the use of NVivo 14 in order to organize and code the data systematically. The coding approach comprised three steps. At first, Open Coding entailed extraction of meaningful units from the interviews, observations, and responses of students on paper and attribution of initial codes to those phenomena which emerged from the data collected. Secondly, Axial Coding entailed analyzing the relationships between the initial codes and putting them into broader categories, especially the kinds of barriers to learning and levels of statistical thinking. Finally, Selective Coding involved synthesis and integration of the core categories in order to build an understanding of the phenomena identified. The matrix coding feature of NVivo 14 allowed analyzing the relationships between the categories.

In order to increase the credibility and trustworthiness of the analysis, the process of coding entailed systematic analysis of the data at the three coding stages. The initial codes were analyzed and refined by comparison of the evidence derived from the interviews, observations, and written responses of the students. NVivo 14 facilitated the organization, retrieval, and comparison of coded data, while the relationships among categories were examined to ensure that the interpretations remained grounded in the collected evidence. As the analysis was conducted by a single researcher, intercoder reliability statistics were not calculated. Instead, analytical consistency was supported through iterative code review, comparison across data sources, and documentation of coding decisions.

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The theoretical framework for analyzing learning obstacles was adapted from the classification of didactical situations developed by Brousseau and Balacheff (2002), which distinguishes learning obstacles into three categories: ontogenetic obstacles, referring to students' psychological and cognitive readiness; didactical obstacles, referring to constraints or difficulties arising from the design and implementation of learning; and epistemological obstacles, referring to limitations in students' conceptual understanding or the applicability of particular mathematical concepts across different contexts. On the other hand, the process of identifying the levels of students' statistical thinking abilities was informed by the taxonomy provided by Jones et al. (2000), that identifies four levels of statistical reasoning, namely: idiosyncratic, transitional, quantitative, and analytical.

Peer debriefing involving the experts in the field of mathematics education was also employed to enhance the credibility of the data by having the coding and interpretations scrutinized. In addition, member checking was carried out among the school authorities relevant to the study to establish the veracity and correctness of the interpretations made from the gathered data (Lincoln, 1985). The process of data collection was discontinued when data saturation was reached, which was marked by the lack of new codes or categories that can be generated from the findings (Glaser & Strauss, 2006).

3. Result and Discussion

Distribution of Students' Learning Obstacles

The analysis carried out through NVivo 14 showed that the learning problems of the students were mostly represented by epistemological obstacles, then by ontogenetic obstacles, and lastly by didactical obstacles. The frequency distribution of the identified learning obstacles based on the indicators of data central tendency is summarized in Table 1.

Table 1. *Distribution of Learning Obstacles by Content Indicator*

Content Indicator	Didactical	Epistemological	Blank Responses	Ontogenetic	Total
Analysis of Measures of Central Tendency	8	33	4	14	59
Mean	0	5	4	10	19
Median	2	10	2	9	23
Mode	0	15	2	7	24
Data Organization	2	11	2	7	22
Total	**12**	**74**	**14**	**47**	**147**

Note. The frequencies represent the number of coded references identified through the NVivo 14 analysis.

As shown from Table 1, epistemological obstacles constituted the most common learning obstacles, contributing 74 codes of learning obstacles, followed by ontogenetic obstacles (47

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codes), blank answers (14 codes), and didactical obstacles (12 codes). The Analysis of Measures of Central Tendency measure had the highest number of epistemological obstacles, constituting 33 codes, followed by Mode (15 codes), Data Organization (11 codes), Median (10 codes), and Mean (5 codes). This pattern shows that students had great difficulties in comprehending and utilizing concepts of measures of central tendency.

The ontogenetic barriers appeared to be most common in the Analysis of Measures of Central Tendency indicator (14 sources), while Mean was the second indicator with the highest number of ontogenetic barriers (10 sources). The results indicate that some students were not sufficiently cognitively ready to combine information provided in the problem with necessary statistical concepts. Didactical barriers occurred rather seldom and were mainly found in the Analysis of Measures of Central Tendency indicator (8 sources). This implies that students' problems were more related to conceptual limitations rather than to instruction-related ones.

Overalll, the results show that the development of the HLT must involve more focus on building up the students' conceptual knowledge about the measures of central tendency. The tasks should allow the students to recognize the features of the problem, choose the measure of central tendency, justify the choice, and interpret the obtained statistical data in accordance with the context. It will help to overcome the main epistemological obstacles that were found in the study and develop the statistical thinking of the students.

Distribution of Students' Statistical Thinking Levels

Students' statistical thinking ability classifications through the NVivo 14 software showed that the coded data was largely clustered in the intermediate category. Students' statistical thinking level distributions are shown in Table 2 below.

Distribution of Students' Statistical Thinking Levels

The distribution of students' statistical thinking levels across the content indicators is presented in Table 2. Classification was done following the scheme developed by Jones et al. (2000), where statistical thinking was divided into four categories: idiosyncratic, transitional, quantitative, and analytical. The number of citations at each category was determined using the NVivo 14 coding method.

Table 2. *Distribution of Statistical Thinking Levels by Content Indicator*

Content Indicator	Analytical	Idiosyncratic	Blank Responses	Quantitative	Transitional	Total
Analysis of Measures of Central Tendency	0	14	5	18	21	58

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Mean	1	5	4	7	8	25
Median	0	3	2	10	10	25
Mode	1	4	2	8	10	25
Data Organization	0	6	2	7	8	23
Total	**2**	**32**	**15**	**50**	**57**	**156**

Note. The frequencies represent coded references identified through the NVivo 14 analysis.

From the results in Table 2, it can be seen that Transitional level is the level that received the highest number of codes, 57 references, while Quantitative level received the second-highest number of codes, 50 references. The results imply that, overall, the students seemed able to apply procedural skills and knowledge related to performing statistical tasks; however, their understanding had not been accompanied by conceptual interpretation. Thus, it can be concluded that the students were relatively good at performing statistical procedures; however, they had problems in relating the procedure with the concepts and meaning of the data used.

Idiosyncratic level received 32 codes; this result shows that some students tend to use intuition or perception without using quantitative data appropriately. It implies that the statistical reasoning of those students had not yet been supported by any relevant quantitative evidence. On the other hand, 15 blank-response references represent students who did not provide any response to the statistical problem.

On the other hand, Analytical level had only 2 coded items, being the level that had the least frequency among all the identified levels of statistical thinking. This implies that a very small number of the coded answers showed higher order thinking in terms of statistics in which there was an analysis and interpretation of the information provided by means of critical evaluation of the data. There is a need for learning activities that will aid the process of moving from computation to interpretation.

On the whole, the distribution shows that the students' reasoning was mainly focused on the transitional and quantitative stages, whereas analytical reasoning remained less developed. Such a distribution of stages is compatible with the learning difficulties highlighted in Table 1, especially epistemological difficulties. Hence, when working on the HLT design, it would be unreasonable to work only on the development of the students' computation skills; instead, students should be provided with an opportunity to elaborate on their thinking and give interpretations of the statistics.

Cross-Analysis of Learning Obstacles and Statistical Thinking Levels

The relation of the groups of learning difficulties and the students' statistical thinking levels was analyzed further using the matrix coding feature in NVivo 14. This cross-analysis was done in order to find out if there are any relations between the types of learning difficulties that the students have and their statistical thinking levels. The findings of the cross-analysis are provided in Table 3 below.

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Relationship Between Learning Obstacles and Students' Statistical Thinking Levels

Table 3 presents the cross-tabulation showing how there is an association between students' learning obstacles (LOs) and statistical thinking level. This is from the matrix coding that was produced by NVivo 14. It has been found to contain some important patterns.

Table 3.
Relationship Between Learning Obstacles and Students' Statistical Thinking Levels

Statistical Thinking Level	Didactical	Epistemological	Blank Responses	Ontogenetic
Analytical	0	1	0	0
Idiosyncratic	4	4	2	23
Blank Response	0	3	8	4
Quantitative	4	35	2	1
Transitional	4	31	2	19
Total	**12**	**74**	**14**	**47**

Note. Values represent coded references identified through matrix coding in NVivo 14.

The matrix in Table 3 highlights the prevalence of ontogenetic obstacles at the Idiosyncratic stage, totaling 23 coded references. This implies that students that highly depended on subjective perceptions and intuitive reasoning also exhibited challenges related to their cognitive readiness, prior knowledge or ability to construct sensible statistical representations.

One of the notable aspects of the results is that epistemological obstacles were most common among students at the Quantitative stage (35 references) and Transitional stage (31 references). This shows that procedural knowledge does not automatically translate into conceptual knowledge of statistical concepts. The ability to apply statistical concepts procedurally does not automatically mean that there is sufficient conceptual understanding. The students may be competent in executing the procedures but hold incomplete and even inappropriate conception of the statistical concepts in question.

The lower number of epistemological obstacles at the Analytical level (1 reference) indicates that statistical reasoning at a more advanced level is related to the increased ability to link procedures with concepts, to critically evaluate information, and to interpret statistical outcomes. The empirical evidence can serve as the background for creating an HLT which takes into account the existence of conceptual obstacles in learning statistics and which develops statistical reasoning step by step.

Transformation of Empirical Findings into HLT Design Recommendations

The observed characteristics of students' errors were subsequently translated into instructional principles for developing a revised Hypothetical Learning Trajectory (HLT). Rather than treating students' incorrect responses merely as assessment outcomes, the identified

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difficulties were interpreted as indicators of specific conceptual, cognitive, and instructional needs. The transformation of these empirical findings into learning implications and HLT design recommendations is presented in Table 4.

Table 4.
Transformation of Empirical Findings into HLT Design Recommendations

Empirical Finding (Students' Test Responses)	Learning Implication	HLT Design Recommendation
Students interpreted the mode as the numerically largest value in a dataset rather than the value occurring most frequently.	Students failed to distinguish between the value representing an object or observation and the frequency with which that value occurs.	**Phase 2 Activity:** Use a concrete block-tower manipulation activity in which students identify the tower with the greatest visual height/frequency, enabling them to construct the meaning of the mode as the most frequently occurring value.
Students mechanically applied the mean formula to datasets containing extreme values (*outliers*).	Students had not yet developed an understanding of the representativeness of a measure of central tendency in relation to the characteristics of the data and its real-world context.	**Phase 3 Activity:** Conduct a simulation comparing employees' wages with the company owner's wage, initially using concrete block-tower representations, to demonstrate how an extreme value can influence the mean.
Students selected one of the two middle values as the median for an even-numbered dataset without calculating their average.	Students demonstrated a distorted conception of the median as a single middle observation rather than a measure that divides an ordered dataset into two balanced parts.	**Phase 2 Activity:** Ask students to physically divide a set of objects into two opposing groups as fairly as possible, leading them to construct the concept of the median for both odd- and even-numbered datasets.
Students obtained correct numerical results but were unable to provide a statistical argument or explain their reasoning.	Students' learning remained predominantly instrumental and procedurally oriented, with limited emphasis on justification and interpretation.	**Phase 4 Activity:** Require students to prepare a mini statistical report and present a justification of their data interpretation, thereby connecting computational procedures with reasoning and communication.
Students left response sheets blank or merely	Students experienced difficulties initiating an	**Phase 1 Activity:** Conduct direct physical data-collection experiments

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copied statements from the problem.	appropriate solution strategy, potentially reflecting limited engagement with the problem and insufficient activation of prior knowledge.	within the classroom environment to increase students' engagement and establish a meaningful experiential context for subsequent statistical reasoning.

Discussion

The NVivo 14 analysis indicates that epistemological obstacles were the most dominant learning obstacles, suggesting that the main challenge in elementary statistics learning lies in students limited conceptual understanding rather than primarily in instructional constraints or didactical obstacles. While the students could apply the procedures and computations correctly, it was difficult for them to justify why the specific procedure had to be applied. This indicates that the students have instrumental knowledge (Skemp, 2013). The students learned the algorithm through memory but lacked the logical and conceptual understanding. This becomes clearer when the students encounter context-based problems where flexibility of thought is needed. The students can see statistics as nothing more than arithmetic calculations and therefore find it hard to relate computation and data. Ontogenetic barriers indicate that the students have inadequate cognitive preparedness to change the story problem into mathematical structure.

The fact that the number of students operating on the transitional and quantitative levels is high implies that there is a proclivity towards mechanical and procedural thinking. While the students on the transitional level acknowledge the significance of data, their reasoning remains intuitive. On the other hand, students on the quantitative level concentrate primarily on getting the right answers without focusing much on interpreting them or understanding what the numbers mean. The low percentage of students on the analytical level shows that the classroom setting does not afford enough opportunity for justifying the ideas, evaluating data, and making decisions regarding whether certain measures of central tendencies are applicable or not. The findings obtained from the matrix coding suggest that students may be skillful quantitatively but lack understanding conceptually. Epistemological obstacles found at the quantitative level imply that the students who are capable of doing calculations related to finding the mean or median may have basic misconceptions of these terms. Conversely, the association between ontogenetic obstacles and the idiosyncratic level highlights the need to strengthen students' cognitive readiness through child-friendly and concrete activities before introducing more formal mathematical representations.

These findings provide an empirical basis for developing a responsive Hypothetical Learning Trajectory (HLT). Given the predominance of epistemological obstacles, the proposed trajectory should avoid introducing formulas as the primary starting point and instead emphasize meaningful experiences that progressively develop conceptual understanding. The resulting four-phase Local Learning Trajectory begins with real-data collection and organization, in which students collect and organize authentic classroom data, such as classmates' heights or shoe sizes. This direct involvement is intended to strengthen engagement and address ontogenetic obstacles. The second

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phase involves visualization of the mean, median, and mode using the block tower representation. In this phase, students will be investigating the mode through frequency and median through balancing ordered data in order to address their misconceptions related to mode and median in an even dataset. The third phase involves the use of datasets with deliberately chosen extremes to create cognitive conflict. By discussion and comparing, students will learn the reasons why the mean is not always a representative description of the dataset, hence enhancing their conceptual understanding. Finally, the fourth phase involves justification and interpretation, where students are expected to go beyond calculations into writing arguments and drawing statistical conclusions.

This study involved the identification of students' learning difficulties and levels of statistical thinking, which in turn led to the development of suggestions for designing an HLT. This designed HLT was neither implemented nor validated by undergoing any form of iterative learning cycles; consequently, there is no evidence that it works. In future studies, it will be important to test out the designed HLT in real classroom settings in order to validate its practicality and effectiveness. This will involve implementing the HLT in classroom settings using design research methods.

4. Conclusion

This study concludes that elementary school students' learning obstacles in mathematics, particularly in the topic of measures of central tendency, were predominantly epistemological obstacles rooted in instrumental and procedural understanding. The statistical thinking of students was mostly confined to the transitional and quantitative stages, while the analytical stage was very poorly developed. The cross-analysis further demonstrated that mathematical proficiency in performing computational procedures does not necessarily eliminate students' conceptual difficulties. In other words, procedural fluency does not automatically lead to conceptual understanding of statistical concepts.

As a response to these findings, the identified learning obstacles and statistical thinking levels were translated into a four-phase Hypothetical Learning Trajectory (HLT). The proposed trajectory progresses from real-data organization and collection, to conceptual meaning-making through concrete visualization, exploration of data characteristics through the manipulation of extreme values, and finally the formulation of independent statistical interpretations and justifications. The proposed HLT provides a potential framework for designing statistics learning that is more conceptual, contextual, and responsive to students' ways of thinking.

However, the proposed HLT remains hypothetical because it has not yet been implemented and evaluated in an actual classroom setting. Thus, its effectiveness in overcoming the learning difficulties of students as well as promoting higher order statistical thinking skills is yet to be proven. It is recommended that future research validate the effectiveness of this framework by implementing and refining it using a design research methodology.

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6. References

Brousseau, G., & Balacheff, N. (2002). *Theory of didactical situations in mathematics: Didactique des mathématiques, 1970–1990*. Kluwer Academic Publishers.

Clements, D. H., & Sarama, J. (2011). Early childhood mathematics intervention. *Science, 333*(6045), 968–970. https://doi.org/10.1126/science.1204537

Creswell, J. W. (2009). *Research design: Qualitative, quantitative, and mixed methods approaches*. SAGE Publications.

Glaser, B. G., & Strauss, A. L. (2006). *The discovery of grounded theory: Strategies for qualitative research*. Aldine Transaction.

Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical Thinking and Learning, 6*(2), 105–128.

Jones, G. A., Thornton, C. A., Langrall, C. W., Mooney, E. S., Perry, B., & Putt, I. J. (2000). A framework for characterizing children's statistical thinking. *Mathematical Thinking and Learning, 2*(4), 269–307. https://doi.org/10.1207/S15327833MTL0204_3

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. SAGE Publications.

Melati, Z., Fuadiah, N. F., & Surmilasari, N. (2024). Analisis hambatan belajar (*learning obstacle*) pada materi statistika siswa kelas VI SDN 6 Palembang. *Pentagon: Jurnal Matematika dan Ilmu Pengetahuan Alam, 2*(3), 136–148. https://doi.org/10.62383/pentagon.v2i3.247

Miles, M. B., Huberman, A. M., & Saldaña, J. (2018). *Qualitative data analysis: A methods sourcebook*. SAGE Publications.

Organisation for Economic Co-operation and Development. (2023). *PISA 2022 results factsheets: Indonesia*. https://oecdch.art/a40de1dbaf/C108

Parra-Fica, J. H., del Mar López-Martín, M., & Díaz-Levicoy, D. (2024). Measures of central tendency in primary education textbooks in Chile. *Eurasia Journal of Mathematics, Science and Technology Education, 20*(6). https://doi.org/10.29333/EJMSTE/14592

Rahayu, S., Masrurah, U., Slamet, S., Murtiyasa, B., & Sumardi, S. (2024). Analisis kesulitan dalam pembelajaran konsep mean, median, dan modus pada peserta didik sekolah dasar. *Pendas: Jurnal Ilmiah Pendidikan Dasar, 9*(3), 106–119.

Sari, H., Pratiwi, M. P., & Syutaridho, S. (2025). Memahami konsep statistika (pemusatan data: Mean, median, dan modus) dalam kehidupan sehari-hari menggunakan pendekatan PMRI. *BLAZE: Jurnal Bahasa dan Sastra dalam Pendidikan Linguistik dan Pengembangan, 3*(2), 132–137.

Skemp, R. R. (2013). Relational understanding and instrumental understanding. *The Arithmetic Teacher, 26*(3), 9–15. http://www.jstor.org/stable/41187667

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Suryadi, D. (2013). *Design research (DDR) dalam pengembangan pembelajaran matematika*.
Utari, R. S., Putri, R. I. I., Zulkardi, Z., & Hapizah, H. (2025). Designing hypothetical learning trajectory in statistical literacy using the local contexts of South Sumatra (pp. 401–416). https://doi.org/10.2991/978-2-38476-390-0_34
Wirayuda, A., Pamungkas, B. A., Marta, I. H., & Muhammad, A. F. N. (2025). Studi literatur: Konsep pengolahan data. *Didaktik: Jurnal Ilmiah PGSD STKIP Subang, 11*(4), 220–232.

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