
Analyzing Students' Thinking Processes in Solving Exponential Problems Based on APOS Theory: A Qualitative Study at SMA Negeri 7 Tanjung Jabung Timur

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Abstract

This study aims to describe students' thinking processes in solving exponential problems based on the APOS theory (action, process, object, and schema). This research employed a descriptive qualitative approach conducted in class X Phase E3 at SMA Negeri 7 Tanjung Jabung Timur. The subjects consisted of three students selected through purposive sampling, representing high, moderate, and low ability levels. Data were collected using written tests in the form of open-ended questions and semi-structured interviews to explore students' thinking processes in depth. The results indicate that students with high ability were able to achieve all stages of APOS, demonstrating a comprehensive understanding and integration of concepts. Students with moderate ability reached the action and process stages adequately but showed limited understanding at the object and schema stages. Meanwhile, students with low ability were only able to perform at the action and partial process stages, relying on procedural steps without deep conceptual understanding. These findings highlight differences in students' conceptual construction across ability levels.

Keywords: The APOS Theory, Student Thinking Processes, Exponents, Mathematics Education

1. Introduction

Mathematics is one of the subjects that plays a crucial role in helping students develop logical, systematic, and critical thinking skills (Anderha & Maskar, 2021). According to Muhammad Sabirin, (2014), mathematics instruction focuses not only on computational skills but also on conceptual understanding and applying these concepts to various problem-solving situations. Therefore, students' thinking processes in understanding mathematical concepts are an important

consideration in the learning process (Khamidah, 2016). However, the reality is that many students still struggle to grasp mathematical concepts in depth, as instruction often places greater emphasis on problem-solving procedures than on conceptual understanding (Fitra et al., 2021). This results in students simply memorizing formulas without understanding the meaning of the concepts they are learning.

One of the math topics that often poses a challenge for students is exponents. The concept of exponents is closely related to various other mathematical ideas, such as logarithms, exponential functions, and exponential growth (Rahma & Khabibah, 2022). According to Mulyani et al., (2022), a lack of understanding of the concept of exponents can lead to errors in applying the properties of exponents and in solving problems involving powers. Several studies indicate that students still face difficulties in understanding the meaning of exponents, particularly regarding zero exponents, negative exponents, and the application of exponential properties in solving mathematical problems. These difficulties often arise because students view exponents merely as repeated multiplication without grasping the underlying concepts Herdian et al., (2019).

In addition, the challenges students face with exponential concepts stem from a lack of foundational understanding, which often leads them to make mistakes when applying principles and procedures to solve problems. According to Nurajijah et al., (2023), students' conceptual understanding of exponential material still needs to be improved through learning methods that can help students build their own understanding of mathematical concepts. One method that can be used to analyze and understand how students think when learning mathematics is the APOS theory (Tatira, 2021).

According to Soku et al., (2025), the APOS (Action, Process, Object, Schema) theory is a constructivist theory developed by Ed Dubinsky and his team to explain how individuals construct their understanding of mathematical concepts. In this theory, mathematical understanding is formed through four main stages: action, process, object, and schema (Widya Pratama, 2022). The action stages occurs when students perform procedures directly based on given instructions or examples. The process stages emerges when students begin to mentally grasp the procedure and can think through the solution stages without always having to follow instructions directly. Then, in the object stages, students can view the process as a mathematical object that can be manipulated. The final stages are the schema, which occurs when students are able to connect several mathematical concepts they have learned into a comprehensive knowledge structure (Yuliana & Ratu, 2018).

According to Yarman et al., (2024) the APOS theory is frequently applied in mathematics education studies to observe how students develop their understanding of mathematics and to identify the challenges students face in learning these concepts. By conducting an analysis based on the APOS stages, educators can understand where students encounter difficulties, thereby enabling them to create more effective lesson plans (Trigueros et al., 2024). Various studies indicate that the use of the APOS theory can help identify errors in students' mathematical concept formation while also providing insight into how students think when solving math problems (Fauziyah & Wahyunita, 2025)

Based on this background, it can be concluded that mathematics learning requires not only procedural skills but also a deep conceptual understanding so that students can think logically, systematically, and critically. However, in reality, many students still struggle to understand concepts, particularly in the area of exponents, because instruction tends to focus on memorizing formulas and procedures. These difficulties highlight the need for a learning approach that helps students build their understanding independently. In this context, the APOS theory (action, process, object, and schema) serves as a relevant framework for analyzing students' thinking processes in understanding mathematical concepts. Through APOS-based analysis, educators can identify the stages of students' difficulties and design more effective instruction to enhance their understanding of mathematical concepts.

2. Methods

This study is a qualitative descriptive study. Qualitative descriptive research aims to describe and deeply understand phenomena occurring in the learning process, particularly students' thinking processes in understanding a mathematical concept. This descriptive process is defined as a detailed and clear depiction of the thinking processes carried out by students (Creswell & David Creswell, 2018). This approach is used to describe how students' thinking processes regarding exponential material are viewed from the stages of theory, action, process, object, and schema.

This study was conducted in Class X, Phase E3, at State Senior High School 7 in Tanjung Jabung Timur, Jambi Province. The research subjects were selected using purposive sampling, meaning they were chosen based on specific criteria relevant to the study's objectives. The research subjects in this study were three students. The selection of research subjects was based on their ability to answer questions on exponential material

The data collection instruments and techniques used in this study were tests and interviews. The tests were used to measure students' ability to solve problems related to exponential concepts, consisting of open-ended questions specifically designed to assess their conceptual understanding of exponents. Additionally, semi-structured interviews were conducted to gain deeper insights into students' thought processes when solving the given problems. The following are the indicators and descriptors from the APOS theory:

Table 1
Indicators and Descriptors of the APOS Theory

No	Indicators	Descriptor
1.	<i>Action</i>	<ol style="list-style-type: none"> 1. Students are able to follow stages-by-stages instructions to solve problems 2. Students are able to apply rules or formulas mechanically 3. Students are able to repeat the procedure to obtain the correct results
2.	<i>Process</i>	<ol style="list-style-type: none"> 1. Students are able to explain the stagess of the solution in their own words.

	2. Students are able to visualize the calculation process without having to write it out in full
	3. Students are able to apply procedures in different situations with minor adjustments
3. <i>Object</i>	1. Students are able to reflect on mathematical concepts as a unified whole
	2. Students are able to connect the concepts they have learned with other concepts
	3. Students are able to use a concept as a foundation for developing new concepts
4. <i>Schema</i>	1. Students are able to integrate various concepts to solve complex problems
	2. Students are able to select and apply appropriate strategies to solve problems
	3. Students are able to revise or expand upon their schemas based on new learning experience

In this study, the researcher employed source triangulation as a method to validate and enrich the data obtained. The research test instrument was validated by a mathematics expert to ensure the accuracy of its construction and the quality of the measurement tool. To strengthen validity, the study will use purposive sampling with specific criteria for selecting research subjects. Subjects were selected to represent the diversity and characteristics of students capable of articulating their thought processes.

Data credibility is achieved through source triangulation by cross-checking test results, interviews, observations, and documentation. This approach involves combining data from various sources, including think-aloud test sheets, recordings of in-depth interviews between the researcher and the subjects, direct observations, and analysis of research documentation.

3.Result and Discussion

The purpose of this study is to describe students' thought processes in solving open-ended problems on exponential material based on the APOS theory indicators, which include action, process, object, and schema. Research data were obtained through written tests and interviews with the research subjects. Subjects were selected using purposive sampling based on specific criteria established by the researcher. Based on the analysis of test answers and interviews, three students were identified who met the research requirements: one with high ability, one with moderate ability, and one with low ability. These three students were then analyzed to determine their thought processes in solving exponential problems based on the stages of the APOS theory. The analysis is presented below:

Advanced-Level Students (ST)

Based on the analysis of ST's answer sheet in Figure 1, it is evident that ST was able to identify key information in the problem and translate it into an exponential mathematical model. The student wrote the production model prior to the fifth month as an exponential function and

recognized that after the fifth month, production conditions changed to 80% of normal levels. This indicates that the student was able to interpret the problem accurately:

Figure 1

Responses from High-Level Students

a. Model matematika
 Sebelum bulan ke-5 $P(n) = 2.500 \times 2^{n-1}$ Setelah bulan ke-5
 Efektif $(n) = 0,8 \times (2.500 \times 2^{n-1})$

b. Produksi bulan ke-4 dan ke-7
 $P(n) = (2.500 \times 2^{(n-1)})$ $P(n) = (2.500 \times 2^{(n-1)})$
 $P(4) = (2.500 \times 2^{(4-1)})$ $P(7) = (2.500 \times 2^{(7-1)})$
 $P(4) = (2.500 \times 2^3)$ $P(7) = (2.500 \times 2^6)$
 $= (2.500 \times 8)$ $= (2.500 \times 64)$
 $= 20.000$ $= 160.000$

$P_{\text{normal}} = 160.000$
 $P_{\text{efektif}} = 0,8 \times 160.000$
 $= 128.000$

c. Perbandingan
 $\frac{P_{\text{efektif}}}{P_{\text{normal}}} = \frac{128.000}{160.000} = 0,8$

Based on Figure 1: High-Level Student Responses, the problem-solving process carried out by students can be analyzed using the APOS theory framework, which consists of the stages of action, process, object, and schema. In the action stage, students were able to follow the problem-solving stages systematically by first writing down the mathematical model, namely $P(n)=2.500 \times 2^{(n-1)}$. Students used this formula correctly to determine production in the 4th month by substituting the value $n = 4$, yielding $P(4) = 2.500 \times 2^3 = 20.000$. Additionally, students were also able to repeat the same procedure when determining the value for the 7th month by applying the correct rules of exponents. The interview is as follows:

P : What was the first thing you did when you read this problem?

ST : I read the problem first, then looked up the given information. From the problem, it's clear that the initial production was 2,500 and the pattern is to double every month, so I used the formula $P(n)=2.500 \times 2^{(n-1)}$.

During the process stage, students demonstrated a good understanding of the stages involved in solving the problem. This was evident when students determined the production for the 7th month by adjusting the formula to $P(7) = 2.500 \times 2^{(7-1)}$, and then performing the calculation to obtain a normal production value of 160,000. Students were also able to apply the same procedure to a different situation, namely when calculating effective production after the 5th month by multiplying the normal production output by 0.8. The interview is as follows:

P : Why did you use the same method to find the value for the 7th month?

ST : Because the pattern is the same, so I used the exponential formula. I just substituted 7 for n , and that gave me $P(7) = 2.500 \times 2^6$.

At the object stage, students have come to view the calculation process as a cohesive concept. Students are able to connect the concept of exponential growth with the concept of percentages to determine effective production. This is evident in the students' stagess, where they distinguish between normal production and effective production, and then use the concept of percentages to calculate the effective production value as $0.8 \times 160,000 = 128,000$.

P : What do you think is the connection between the exponential calculation and the 80% you used?

ST : The exponential is used to find the normal monthly production volume, while the 80% is used to calculate the actual production after the reduction. So the result of the exponential calculation serves as the basis for calculating the final value.

Next, in the schema stage, students are able to integrate various mathematical concepts used to solve problems comprehensively. Students combine the concepts of exponential patterns, substituting values into formulas, and percentages to compare normal production and effective production. In addition, students also demonstrate the ability to choose the correct solution stagess by presenting the final result in the form of a comparison between effective production and normal production, resulting in a ratio of $\frac{128,000}{160,000} = 0,8$. This indicates that students have a well-organized

understanding of how to solve the given problem.

P : So what's the conclusion of your calculations?

ST : The conclusion is, ma'am, that normal production for the 7th month is 160,000 units, and effective production is 80% of that, which comes out to 128,000. That gives us a ratio of 0.8.

Intermediate-Level Students (SS)

Based on the analysis of the SS answer sheet in Figure 2, subjects in the moderate ability category were able to complete most of the stagess involved in the mask production problem, but still showed inconsistencies in the accuracy of their calculations and the use of mathematical notation. See Figure 2 below:

Figure 2
Answers from Intermediate-Level Students

a. masker
 $R_n = 2.500 \times 2^{n-1}$
 Setelah Bulan ke-5
 $P_{\text{Efektif}}(n) = 0,8 \times 2.500 \times 2^{n-1}$

b. masker bulan ke-4 dan bulan ke-7
 bulan ke-4
 $P(4) = 2.500 \times 2^3 = 20.000$
 bulan ke-7
 $P(7) = 2.500 \times 2^6 = 160.000$
 $P_{\text{Efektif}}(7) = 0,8 \times 160.000 = 128.000$

c. Perbandingan
 Penurunan Produksi = $160.000 - 128.000 = 32.000$

Based on Figure 2: Answers from Intermediate-Level Students, the problem-solving process carried out by students can be analyzed according to the stages in the APOS theory, namely action, process, object, and schema. In the action stage, students were able to follow basic instructions in solving the problem by writing the mathematical model $P(n) = 2.500 \times 2^{n-1}$ and writing the effective production after the 5th month, namely $P_{efektif}(n) = 0,8 \times 2.500 \times 2^{n-1}$. This indicates that students can mechanically apply the formulas they have learned and perform basic calculation procedures, such as determining the production in the 4th month by calculating $2.500 \times 2^3 = 20.000$. The interview is as follows:

P : What was the first thing you did when you read this problem?

SS : I read the problem and then wrote down the formula from the material, which is

$$P(n) = 2.500 \times 2^{n-1}.$$

In the process stage, students begin to apply the same procedure to determine production in the 7th month by substituting the value of n ke into the formula, resulting in $P(7) = 2.500 \times 2^6 = 160.000$. Students then adjust the calculation to the conditions in the problem, namely by multiplying the result by 0.8 to obtain an effective production of 128,000. This demonstrates that students are able to apply the same procedure to different situations, although they are still limited to the stages that were previously demonstrated

P : How did you calculate the production for the 7th month?

SS : I used the same method as before, just substituting 7 for n , so 2.500×2^6 .

At the “object” stage, students begin to understand that the result of an exponential calculation represents normal production, which is then used as the basis for determining effective production. Students are able to connect the concept of exponential growth with the concept of percentage reduction in production. However, this understanding is not yet fully organized because at the end, students only write down the difference between normal production and effective production, namely $160.000 - 128.000 = 32.000$, without providing further explanation regarding the meaning of the comparison requested in the question.

P : Why are you calculating the difference between 160,000 and 128,000?

SS : I want to see how much production has decreased after a 20% reduction.

At the schema stage, students have attempted to integrate several concepts used in problem-solving, namely the concepts of exponents and percentages. However, this integration is not yet fully optimal because students have not fully developed a comprehensive problem-solving schema to explain the relationship between normal production and effective production. This indicates that students’ understanding is already at the stage of combining several concepts, but they still need reinforcement in organizing these concepts more comprehensively

P : What is the conclusion of your calculations?

SS : Normal production in the 7th month is 160,000; after the reduction, it becomes 128,000, so there is a decrease of 32,000.

Lower-grade students (SR)

Based on an analysis of the written responses, students in the low-ability category have not yet been fully able to address the challenges of mask production in a comprehensive and systematic manner. See Figure 3 below:

Figure 3
Answers from Lower-Level Students

Handwritten mathematical work showing calculations for mask production over time. The work is organized into three parts: a, b, and c.

Part a: bulan ke-5
 $P(n) = 2.500 \times 2^{n-1}$
 Setelah bulan ke-5
 Pelektif (n) = $0,8 \times (2.500 \times 2^{n-1})$, $n > 5$

Part b: bulan ke-4
 $P(4) = 2.500 \times 2^{4-1}$
 $= 2.500 \times 2^3$
 $= 2.500 \times 8$
 $= 20.000$

Part c: bulan ke-7
 $P(7) = 2.500 \times 2^{7-1}$
 $= 2.500 \times 2^6$
 $= 2.500 \times 64$
 $= 160.000 \times 0,8$
 $= 128.000$

Part d:
 Proforma (n) = 160.000
 Pelektif (n) = 128.000

Based on Figure 3: Low-Level Student Responses, students' problem-solving processes can be analyzed using the stages of the APOS theory: action, process, object, and schema. In the action stage, students were already able to follow the basic stages in solving the problem by writing down the mathematical model used, namely $P(n) = 2.500 \times 2^{n-1}$ as well as writing down the effective production after the 5th month, which is $0,8 \times (2.500 \times 2^{n-1})$. This indicates that students can apply the formulas they have learned and follow the given procedures. In addition, students also attempted to calculate the value for the 4th month by substituting $n = 4$ into the formula, yielding a result of 20,000. However, the stages taken were still mechanical and followed the example without a clear explanation of the rationale behind those stages. The interview is as follows:

P : How did you calculate the value for the 4th month?

SR : I substituted 4 for n in the formula, then I calculated $2,500 \times 2^3$ and the result was 20,000.

During the process stage, students attempted to apply the same procedure to determine the value for the 7th month by calculating $P(7) = 2,500 \times 2^6$, resulting in a value of 160,000. After that, students multiply the result by 0.8 to determine the effective production of 128,000. However, the stages taken by students are still limited to the calculation process and are not accompanied by an explanation of the relationship between the stages used

P : How did you calculate the production for the 7th month?

SR : I used the same formula, but replaced the value of n with 7, so it became $2,500 \times 2^6$.

At the object stage, students' understanding of the concepts involved is still limited. Students have not yet fully grasped the concepts of exponents and percentages as a single, interconnected concept. This is evident in the way students simply write down the normal yield and effective yield without providing further explanation regarding the relationship between these two values. The following is an excerpt from an interview:

P : What do you understand about the numbers 160,000 and 128,000 that you wrote down?
SR : 160,000 is the result of the formula you gave me, and 128,000 is the result after multiplying by 0.8.

At the schema stage, students are not yet fully capable of comprehensively integrating the various concepts used in problem-solving. Students only wrote down the final result of normal production at 160,000 and effective production at 128,000 without providing an explanation that demonstrates the relationship between the concepts of exponents and percentages within a coherent framework of understanding. This indicates that students' understanding is still limited to the application of basic procedures and has not yet developed into a well-organized conceptual schema. The following is the interview:

P : What conclusion can you draw from the answers you obtained?
SR : Production in the 7th month was 160,000, and then it decreased to 128,000 units.

Based on the analysis, during the action phase, the three subjects (ST, SS, and SR) demonstrated relatively similar initial abilities, namely the ability to identify key information in the problem and write the exponential mathematical model $P(n) = 2,500 \times 2^{(n-1)}$. This aligns with research EK Ajeng Rahmi Pinahayu, (2015) showing that students in all three categories can use the formulas they have learned to begin the problem-solving process. However, there are differences in the accuracy and understanding of the procedures performed (Robiah Nofikusumawati Peni, 2025). According to Febrila et al., (2023), students were able to use the formulas they had learned as the first step in problem-solving. However, the difference lies in the quality of understanding. ST students were able to use the formulas systematically and consistently; SS students tended to follow procedures based on examples they had studied; whereas SR students still applied them mechanically without understanding the rationale behind the formulas. This finding aligns with Borji et al., (2024), which states that the APOS theory's action stage is the initial phase when students perform transformations based on learned rules.

During the process stage, differences in ability became more apparent. ST was able to explain the solution steps in a logical sequence, including understanding that the exponential result represents normal production, which is then used to calculate effective production by multiplying by 80%. According to Rosyidi & Hasanah, (2022), high-level students are able to explain the solution steps in a coherent manner and understand that the exponential formula is used to determine normal production before calculating effective production. Furthermore, according to Widya Pratama, (2022), SS students are also able to follow the same procedure to calculate production in the 7th month and effective production, but their explanations remain limited to procedural steps. Meanwhile, Yarman et al., (2024b) found that SR students only performed calculations without

being able to explain the relationships between the steps used. This indicates that ST students have achieved a more mature understanding of the process, while SS students are still in a transitional stage, and SR students remain at a basic procedural understanding.

At the object stage, ST has been able to view the calculation process as a unified concept. ST can connect the concept of exponential growth with the concept of percentages to determine effective production and understand the meaning of the results obtained. This is consistent with research Herdian et al., (2019) stating that high-level students have been able to view the calculation process as a unified concept. Students can connect the concept of exponential growth with the concept of percentage to determine effective production. Meanwhile, SS began to demonstrate an understanding of this relationship but was not yet able to organize it comprehensively. According to Umam & Susandi, (2022), intermediate-level students begin to demonstrate an understanding of the relationship between normal production and effective production, but this understanding is not yet fully organized. This is evident from SS's answers, which focused only on the difference in results without explaining the meaning of the requested comparison. On the other hand, SR has not yet demonstrated a strong conceptual understanding, as they only wrote down the final result without explaining the relationship between the concepts of exponents and percentages. In line with the research Borji et al., (2024), it is stated that lower-level students have not yet fully understood the relationship between the concepts of exponents and percentages. Students only wrote down the final results without providing an explanation of the relationship between the two concepts. Thus, only ST has optimally reached the object stage

Furthermore, at the schema stage, ST is able to comprehensively integrate various mathematical concepts, namely exponents, value substitution, and percentages. This aligns with research Antasari et al., (2023) indicating that high-achieving students are capable of integrating various concepts used in problem-solving, namely exponents, value substitution, and percentages. Therefore, ST is also able to explain the relationship between normal production and effective production systematically, even presenting a comparison of results as a form of comprehensive understanding (Fauziyah & Wahyunita, 2025). Meanwhile, SS has attempted to connect several concepts, but the integration is not yet complete because they have not been able to construct a comprehensive understanding structure. This is consistent with research Yuliana & Ratu, (2018) indicating that intermediate-level students have attempted to connect some of the concepts used, but the integration of these concepts is not yet fully optimal. Intermediate-level students still require assistance or reinforcement to develop a more comprehensive understanding (Longe & Maharaj, 2023). Meanwhile, SR students have not yet been able to integrate the concepts used, so their solutions remain limited to basic calculations without demonstrating relationships between concepts. According to Soku et al., (2025), the solutions produced are still limited to basic computational steps without demonstrating relationships between the concepts used. According to (Winarsih et al., 2019), the schema stage is the highest stage in the APOS theory, where students are able to organize various mathematical concepts into an integrated understanding structure to solve more complex problems.

The overall analysis above shows that differences in students' ability levels are significantly influenced by the depth of mental construction at each stage of the APOS model. ST was able to

fully achieve all stages, from action to schema; SS was at the process stage and beginning to move toward the object stage; while SR was still predominantly at the action stage and partially at the process stage. This indicates that understanding mathematical concepts is determined not only by the ability to perform calculations but also by the ability to connect and integrate mathematical concepts comprehensively. These findings reinforce the view that mathematics instruction should emphasize the development of conceptual understanding so that students can reach the schema stage in APOS theory.

4. Conclusion

Based on the analysis of students' written responses and interviews, this study reveals differences in students' thinking processes in solving exponential problems according to the stages of APOS theory. High-ability students were able to achieve all APOS stages, from action to schema, and demonstrated the ability to integrate exponential and percentage concepts comprehensively. Moderate-ability students successfully reached the action and process stages but showed limited development at the object and schema stages. Meanwhile, low-ability students were only able to perform at the action and partial process stages, relying mainly on procedural steps without demonstrating a coherent conceptual understanding. This study has several limitations, including the small number of participants (only three students), the focus on a single school, and the limitation to one mathematical topic (exponents), which restricts the generalizability of the findings.

Therefore, future research is recommended to involve a larger and more diverse sample, include multiple schools, and explore other mathematical topics. Additionally, further studies could develop intervention-based research using APOS theory to enhance students' conceptual understanding more effectively.

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