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Markov Chain Model for Daily Rainfall Modeling in Bengkulu City

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Abstract

Bengkulu City is a region in Indonesia that is particularly vulnerable to shifts in rainfall patterns, which can have significant impacts on the agricultural sector, water resource management, and disaster mitigation. The uncertainty in rainfall patterns often complicates long-term planning. Hence, it is necessary to adopt a statistical approach that can model and predict rainfall characteristics with greater accuracy. This research aims to develop a Markov Chain model to represent the daily rainfall regime in Bengkulu City. The daily rainfall data are categorized into rainfall intensity states, namely: no rain, light, moderate, heavy, or very heavy rainfall. By leveraging historical daily rainfall data, this model is expected to identify the transition probabilities between these states. Based on the obtained steady-state probabilities, it can be concluded that regardless of today's rainfall condition in Bengkulu City, the long-term probabilities for the weather are as follows: 38% for no rain, 43% for light rain, 13.8% for moderate rain, 4.2% for heavy rain, and 1% for very heavy rain.

Keywords: Markov Chains, rainfall prediction, stochastic, weather

1. Introduction

Rainfall is one of the main components of the climate system that significantly influences various sectors, particularly agriculture, water resources, and the mitigation of hydrometeorological disasters such as floods and landslides (Elkenawy et al., 2024). Rainfall refers to the amount, intensity, duration, and spatial distribution of liquid precipitation resulting from the condensation of atmospheric water vapor within clouds, typically measured in units of depth (e.g., millimeters) over a specified time period and area (Dunkerley, 2023). One millimeter of rainfall corresponds to one liter of water falling on each square meter of area (1 L/m²) (Dunkerley, 2023). According to the Meteorology, Climatology, and Geophysics Agency (BMKG), rainfall is classified into four categories: light rain (1–20 mm/day), moderate rain (20–50 mm/day), heavy rain (50–100 mm/day), and very heavy rain (greater than 100 mm/day).

In a given region, rainfall intensity varies periodically due to atmospheric and climatic factors. In this context, weather monitoring instruments play a crucial role in providing accurate meteorological data for climate analysis and forecasting (Putra et al., 2024). Furthermore, understanding rainfall characteristics is essential for human survival, particularly in relation to food security, water availability, and disaster risk reduction (Elkenawy et al., 2024). Therefore, a system capable of forecasting rainfall using accumulated meteorological data is required. Anticipating potential events or disasters caused by extreme weather conditions can be supported by reliable forecasting systems and the meteorological data currently available (Putra et al., 2024).

In Bengkulu City, climate change—characterized by increasing instability in rainfall patterns—has posed growing challenges for agricultural planning, water resource management, and disaster mitigation. Moreover, the geographical condition of Bengkulu, which lies along the western coast of Sumatra Island, contributes to high and fluctuating rainfall intensity throughout the year (Maharani et al., 2025). Such coastal tropical regions are widely recognized as areas with complex rainfall variability influenced by atmospheric circulation and ocean–atmosphere interactions (Elkenawy et al., 2024).

A predictive model for rainfall is crucial to assist stakeholders in planning and risk mitigation efforts. One effective statistical approach for modeling discrete weather events is the Markov Chain model. This model enables the analysis of transition probabilities between weather states, such as rainy and non-rainy conditions, based on the weather status of the previous day. By utilizing the Markov Chain model, information on rainfall patterns can be transformed into probabilistic events, thereby facilitating short-term rainfall forecasting and supporting risk-based decision-making (Elkenawy et al., 2024).

Various previous studies have demonstrated the reliability of the Markov Chain model in predicting weather transition probabilities and rainfall patterns across different regions. In Indonesia, several studies have successfully applied Markov Chain models to analyze rainfall characteristics and transition probabilities between different rainfall states. For example, a study conducted in Semarang City showed that the Markov Chain model can effectively estimate the probability of transitions between different rainfall intensity states and identify steady-state rainfall patterns useful for flood risk anticipation (Tsani et al., 2024). Similarly, research in Makassar applied a Markov Chain model with an empirical Bayesian approach to describe rainfall characteristics such as transition probabilities, duration of wet and dry periods, and recurrence intervals of rainfall events (Sanusi, 2024). Studies in other Indonesian regions, such as Gorontalo, have also demonstrated that rainfall conditions can be effectively modeled using discrete states (dry, moist, and wet) within a Markov Chain framework to analyze rainfall persistence and transition behavior (Nasib et al., 2022).

Further research conducted in Padang indicated that the Markov Chain method can be used to analyze rainfall patterns and estimate the probability of future rainfall conditions based on historical rainfall data obtained from BMKG stations (Arshinta & Ahmad, 2019). In Ambon City, rainfall prediction using Markov Chain modeling also demonstrated stable transition patterns among rainfall intensity categories, which can support decision-making for fisheries and climate adaptation planning (Rumeon et al., 2025)

Although these studies confirm the usefulness of Markov Chain approaches for rainfall modeling in several Indonesian regions, most existing research still focuses on specific cities or monthly rainfall datasets. Comprehensive stochastic modeling of daily rainfall patterns in coastal regions along the western coast of Sumatra, such as Bengkulu City, which is strongly influenced by atmospheric circulation from the Indian Ocean, remains limited in the existing literature. Therefore, this study aims to address this gap by applying a five-state Markov Chain model (no rain, light rain, moderate rain, heavy rain, and very heavy rain) to represent the daily rainfall dynamics associated with the microclimate characteristics of Bengkulu City. This study differs from previous research by incorporating daily rainfall data in Bengkulu City, the model includes a 5-state model, and followed by validation approach.

This study focuses on the development of a Markov Chain model to represent the daily rainfall pattern in Bengkulu City. The modeling of transition probabilities for daily rainfall intensity is formulated using a first-order Markov Chain.

The selection of a first-order model is predicated on the principle of parsimony to ensure the reliability of the probability estimations. Given that the observations comprise daily data over a five-year period (2020–2024), increasing the model to a second order or higher would exponentially expand the state space dimensions. This expansion could lead to a sparse matrix phenomenon, wherein numerous possible transitions between intensity states lack sufficient empirical frequency, thereby compromising the model's validity. Furthermore, a first-order model is deemed computationally adequate to capture the short-term weather dependencies that represent a dominant characteristic of daily rainfall fluctuations. The results are expected to serve as an initial step toward establishing a statistically based rainfall prediction system for the region, contributing to improved climate adaptation and disaster preparedness strategies.

2. Methods

2.1 Data Source and Research Variables

The data used in this study consist of daily rainfall measurements recorded once every 24 hours using a rain gauge and expressed in millimeters (mm). The daily rainfall data were obtained from the Meteorology, Climatology, and Geophysics Agency (BMKG). Specifically, this study utilized rainfall data from Bengkulu City, which were acquired from the BMKG Meteorological Station in Bengkulu through the official BMKG online database (*Data Online - Direktorat Data Dan Komputasi BMKG*, n.d.). The dataset contains the observation dates along with the corresponding daily rainfall amounts recorded on those dates. The data cover a five-year period, from 2020 to 2024, consisting of a total of 1,850 observations. Table 1 presents a summary of the obtained rainfall data.

Table 1. Research Data

Date	Average	State
1/1/2020	65.9	Heavy rain
1/2/2020	11.7	Light rain
1/3/2020	19.65	Light rain
1/4/2020	1	Light rain
1/5/2020	0	No rain
1/6/2020	0.33	No rain
1/7/2020	0	No rain
1/8/2020	15.3	Light rain
1/9/2020	17.8	Light rain
1/10/2020	31.7	Moderate rain
1/11/2020	29.5	Moderate rain
1/12/2020	10	Light rain
1/13/2020	32.5	Moderate rain
1/14/2020	15.18	Light rain
…	…	…
12/31/2024	1.33	Light rain

The daily rainfall data presented in Table 1 are expressed in millimeters (mm) and are categorized according to daily rainfall intensity. The classification of rainfall intensity follows the standard defined by the Meteorology, Climatology, and Geophysics Agency (BMKG), which consists of five categories: no rain (0 mm/day), light rain (1–20 mm/day), moderate rain (20–50 mm/day), heavy rain (50–100 mm/day), and very heavy rain (>100 mm/day) (Sabrina et al., 2021). For example, during period 1, the recorded rainfall intensity was 65.9 mm, which is classified as heavy rain, whereas in period 2, the rainfall intensity was 11.7 mm, corresponding to the light rain category.

For analytical convenience, each rainfall category is represented as a discrete state, namely state 0 for no rain, state 1 for light rain, state 2 for moderate rain, state 3 for heavy rain, and state 4 for very heavy rain. This state-based classification is commonly adopted in rainfall modeling studies using Markov Chain approaches, where weather conditions are represented as transitions between discrete states over time.

Based on this classification, the total number of observation days in each rainfall category is 701 days with no rain, 795 days with light rain, 255 days with moderate rain, 79 days with heavy rain, and 19 days with very heavy rain.

2.2 Data Analysis Technique

The data analysis technique used to examine the rainfall pattern in Bengkulu City is the Markov Chain model. The following are the steps of the analysis, as adapted from the references (Karlin & Taylor, 2012)

1. Conduct descriptive statistical analysis and data plotting.
2. Categorize the daily rainfall data in Bengkulu City based on the intensity of rainfall recorded each day, where the categories are labeled as follows: “No rain” = 0, “Light rain” = 1, “Moderate rain” = 2, “Heavy rain” = 3, “Very heavy rain” = 4,
3. Construct a state transition table, which contains the number of transitions from state i to state j.

Table 2. Number of Transition between state

Transition	Number of Transition	Transition	Number of Transition		Transition	Number of Transition
$0 \rightarrow 0$	a	$1 \rightarrow 0$	d	…	$m \rightarrow 0$	g
$0 \rightarrow 1$	b	$1 \rightarrow 1$	e	…	$m \rightarrow 1$	h
$0 \rightarrow m$		$1 \rightarrow m$		…	$m \rightarrow m$	
Total	c		f	…		l

The transition 0 → 0 indicates that, within a sequence of events, a transition occurs from state 0 to state 0. All such occurrences are counted and recorded in the table as a, and the same procedure applies to the other transitions accordingly.

4. Calculating the Transition Probability Matrix

In a Markov process, the transition probability matrix represents the likelihood of transitioning from one state to another (Pinsky et al., 2011). The probability that X_{t+1} is in state j given that X_t is in state i is referred to as the one-step transition probability and is denoted by $P_{ij}^{t,t+1}$. This can be expressed as follows:

$$P_{ij}^{t,t+1} = P(X_{t+1} = j \mid X_t = i)$$

A Markov chain is said to have **stationary (or time-homogeneous) transition probabilities** when the one-step transition probabilities do not depend on the time variable t. In such a case,

$$P_{ij}^{t,t+1} = P_{ij}$$

and P_{ij} represents the conditional probability that the system transitions from state i to state j in one step. Typically, these transition probabilities are arranged in the form of a transition probability matrix, as shown below:

$$P = \begin{bmatrix} P_{00} & P_{01} & P_{02} & \cdots & P_{0\mathrm{m}} \\ P_{10} & P_{11} & P_{12} & \cdots & P_{1\mathrm{m}} \\ P_{20} & P_{21} & P_{22} & \cdots & P_{2\mathrm{m}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ P_{\mathrm{m}0} & P_{\mathrm{m}1} & P_{\mathrm{m}2} & \cdots & P_{\mathrm{mm}} \end{bmatrix}$$

A Markov matrix, also known as a transition probability matrix, represents the transition probabilities of a stochastic process and is denoted by $\mathbf{P} = (P_{ij})$.

Each element P_{ij} of the matrix must satisfy the following conditions:

a. $0 \leq P_{ij} \leq 1$, for all $i, j = 0, 1, 2, \dots, m$;

b. $\sum_{j=0}^{m} P_{ij} = 1, for\ all\ i, j = 0, 1, 2, \dots, m,$

which ensures that the total probability of transitioning from state $i$to all possible states equals one.

To construct the transition probability matrix, let n_i denote the total number of observations in which the process occupies state i and let n_{ij} represent the number of transitions from state i to state j within one transition step. Given a sequence of observed transitions among states, the transition probability P_{ij} can be estimated using the following empirical estimator:

$$P_{ij} = \frac{n_{ij}}{n_i}, i, j = 0, 1, \ldots, m,$$

where:
P_{ij} is the transition probability from state i to state j;
n_{ij} is the number of observed transitions from state i to state j;
n_i is the total number of transitions originating from state i.

Based on the transition counts presented in Table 2, the transition probability matrix is estimated for each observation period. For example, if the number of transitions from state 0 to state 0 ($0 \rightarrow 0$) is denoted by n_{00}, then the corresponding transition probability is calculated as

$$P_{00} = \frac{n_{00}}{n_0},$$

where P_{00} represents the probability of remaining in state 0, and n_0 denotes the total number of transitions originating from state 0.

5. Steady-State Probability

The steady-state probability vector, denoted by $\boldsymbol{\pi} = (\pi_0, \pi_1, \pi_2, \ldots, \pi_m)$, can be obtained by solving the stationary distribution equation of the Markov process (Hillier & Lieberman, 2010). The steady-state probabilities describe a condition in which the probability of the process being in each state remains constant over time.

The steady-state distribution may be reached after the process evolves for a sufficiently large number of steps. The n-step transition probability matrix, $\mathbf{P}^{(n)}$, is obtained by raising the one-step transition matrix $\mathbf{P}$to the n-th power. In the long run, the transition probabilities converge such that

$$\lim_{n \to \infty} P_{ij}^{(n)} = \pi_j > 0,$$

where the steady-state probability π_j satisfies the following balance equation:

$$\pi_j = \sum_{i=0}^{m} \pi_i P_{ij}\, , j = 0,1, \dots , m,$$

subject to the normalization condition

$$\sum_{j=0}^{m} \pi_j = 1$$

3. Result and Discussion

3.1 Statistic Descriptive

The descriptive statistics of the rainfall data for Bengkulu City are presented in Table 3.

Table 3. Descriptive Statistics of Rainfall Data in Bengkulu City, 2020–2024

N	Min	Max	Median	Mean	Standard Deviation
1849	0	270	3.4	11.927	21.06

The results indicate that the maximum daily rainfall recorded in Bengkulu City during the study period was 270 mm/day. This value falls into the category of very heavy rainfall, as it exceeds 100 mm/day. In contrast, the mean daily rainfall is 11.927 mm/day, which corresponds to the light rainfall category. The median value of 3.4 mm/day further suggests that most rainfall events during the period were of low intensity. Additionally, the relatively large standard deviation (21.06 mm/day) indicates substantial variability in daily rainfall, reflecting a highly heterogeneous rainfall pattern in Bengkulu City over the study period.

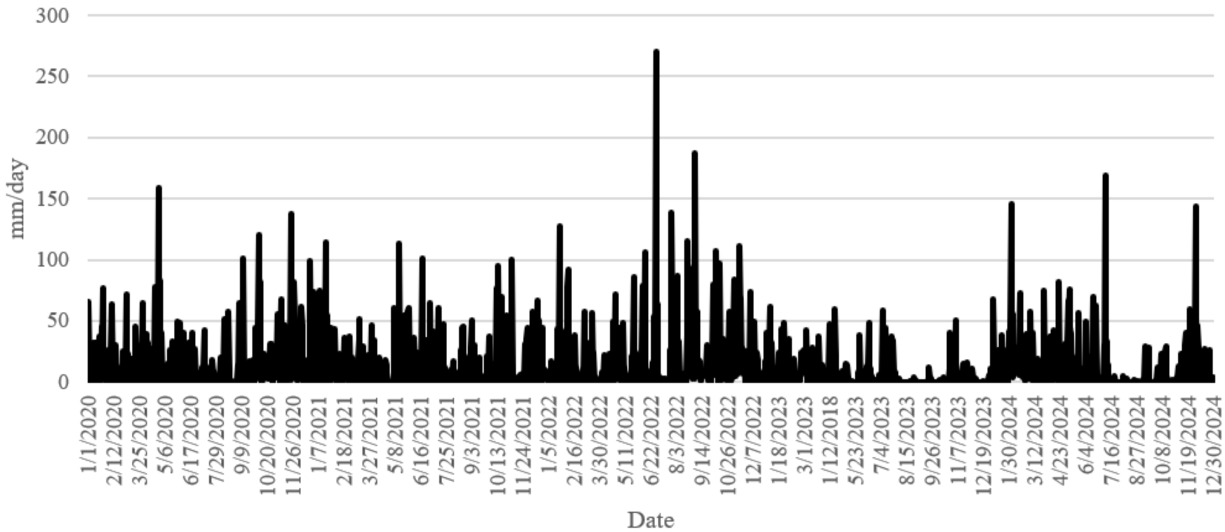

Figure 1. Daily Rainfall in Bengkulu

Figure 1 illustrates the daily rainfall pattern in Bengkulu over the observation period. The figure indicates that the highest daily rainfall was recorded in June 2022. In addition, the graph shows the occurrence of several days with zero rainfall, reflecting the presence of dry days within the study period.

3.2 Rainfall data classification

Based on the classification criteria established by the BMKG, rainfall intensity is categorized into five discrete states (Sabrina et al., 2021), labeled from 0 to 4. State 0 corresponds to days with no rainfall, state 1 represents light rainfall, state 2 indicates moderate rainfall, state 3 denotes heavy rainfall, and state 4 represents very heavy rainfall. Using this classification scheme, the daily rainfall data for Bengkulu were transformed into categorical states, and the frequency of each rainfall intensity state was subsequently determined, as presented in Table 4.

Table 4. Number of days of each state

State	Category	Number of days	Percentage
0	No rain	701	37.89
1	Light rain	795	42.97
2	Moderate rain	255	13.78
3	Heavy rain	79	4.27
4	Very heavy rain	19	1.03

Table 4 summarizes the number of days and corresponding percentages for each rainfall intensity state in Bengkulu during the study period. The results indicate that state 1 (light rainfall) is the most dominant category, accounting for 795 days, or approximately 42.97% of the total observations. This finding suggests that nearly half of the days in Bengkulu are characterized by light rainfall conditions, highlighting the prevalence of low-intensity rainfall in the region.

In contrast, state 0 (no rainfall) comprises 701 days, representing 37.89% of the total, indicating that dry days also constitute a substantial proportion of the observed period. Moderate rainfall (state 2) occurs less frequently, with 255 days or 13.78%, while heavy (state 3) and very heavy rainfall (state 4) events are relatively rare, accounting for only 4.27% and 1.03% of the days, respectively.

Overall, the distribution of rainfall intensity states demonstrates that Bengkulu's rainfall pattern is dominated by light rainfall and dry days, whereas extreme rainfall events occur infrequently.

3.3 One-step transition probability matrix

To construct the one-step transition probability matrix, the rainfall intensity for each day is first classified into one of five discrete states, namely states 0, 1, 2, 3, or 4, according to the established rainfall intensity categories. Subsequently, the transitions between states are identified by observing the changes in rainfall intensity between consecutive days. For each pair of successive days, the rainfall state on the current day is recorded as the initial state, while the rainfall state on the following day is recorded as the subsequent state.

Using this procedure, the observed transitions between rainfall intensity states in Bengkulu City during the period 2020–2024 are summarized in Table 5. Note that the total number of transition events recorded in the frequency matrix is 701 + 794 + 255 + 79 + 19 = 1,848. This figure represents the total number of daily observations (1,849 days) minus one (N−1), which is a mathematical consequence of forming state pairs from time t to t+1

Table 5 The number of transitions between rainfall intensity states in Bengkulu during the period 2020–2024.

Transition	Number of Transition	Transition	Number of Transition	Transition	Number of Transition	Transition	Number of Transition	Transition	Number of Transition
0 → 0	393	1 → 0	233	2 → 0	53	3 → 0	17	4 → 0	5

0 → 1	235	1 → 1	401	2 → 1	114	3 → 1	36	4 → 1	9
0 → 2	53	1 → 2	122	2 → 2	61	3 → 2	15	4 → 2	4
0 → 3	18	1 → 3	30	2 → 3	19	3 → 3	10	4 → 3	1
0 → 4	2	1 → 4	8	2 → 4	8	3 → 4	1	4 → 4	0
Total	701	Total	794	Total	255	Total	79	Total	19

Each transition represents the change in rainfall intensity from one day (current state) to the following day (next state), where the states are defined according to the BMKG classification. The rows correspond to the current day's rainfall state, while the columns represent the rainfall state on the subsequent day.

The table shows that transitions tend to occur more frequently among lower rainfall intensity states. For example, transitions from state 0 (no rainfall) to state 0 and from state 1 (light rainfall) to state 1 have the highest frequencies, indicating a strong persistence of dry and light rainfall conditions from one day to the next. In contrast, transitions involving heavy (state 3) and very heavy rainfall (state 4) occur relatively infrequently, reflecting the rarity of extreme rainfall events in Bengkulu.

To construct the one-step transition probability matrix, each transition count is normalized by the total number of transitions originating from the corresponding current state. Specifically, the transition probability from state i to state j, denoted by P_{ij} is calculated as $P_{ij} = n_{ij}/n_i$

where n_{ij} represents the number of observed transitions from state i to state j, and n_i denotes the total number of transitions originating from state i.

As an illustration, the probability that a day with no rainfall (state 0) is followed by another day with no rainfall (state 0 → 0) is calculated as

$$P_{00} = \frac{n_{00}}{n_0} = \frac{393}{701} = 0.5606.$$

Thus, the element P_{00} of the transition probability matrix equals 0.5606, indicating that more than half of the days without rainfall are followed by another dry day.

Similarly, the probability that a day with light rainfall (state 1) is followed by a day with no rainfall (state 1 → 0) is given by

$$P_{10} = \frac{n_{10}}{n_1} = \frac{233}{794} = 0.2935.$$

This result implies that approximately 29.35% of days with light rainfall are followed by dry conditions on the next day. The remaining elements of the one-step transition probability matrix are computed using the same procedure for all state-to-state transitions. Consequently, a 5 × 5 transition probability matrix is obtained, as follows:

$$P = \begin{bmatrix} P_{00} & P_{01} & P_{02} & P_{03} & P_{04} \\ P_{10} & P_{11} & P_{12} & P_{13} & P_{14} \\ P_{20} & P_{21} & P_{22} & P_{23} & P_{24} \\ P_{30} & P_{31} & P_{32} & P_{33} & P_{34} \\ P_{40} & P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix} = \begin{bmatrix} 0.5606 & 0.3352 & 0.0756 & 0.0257 & 0.0029 \\ 0.2935 & 0.5050 & 0.1537 & 0.0378 & 0.0101 \\ 0.2078 & 0.4471 & 0.2392 & 0.0745 & 0.0314 \\ 0.2152 & 0.4557 & 0.1899 & 0.1266 & 0.0127 \\ 0.2632 & 0.4737 & 0.2105 & 0.0526 & 0.0000 \end{bmatrix}$$

From this matrix, the Markov chain pattern of rainfall in Bengkulu City can be constructed and represented in the following transition diagram.

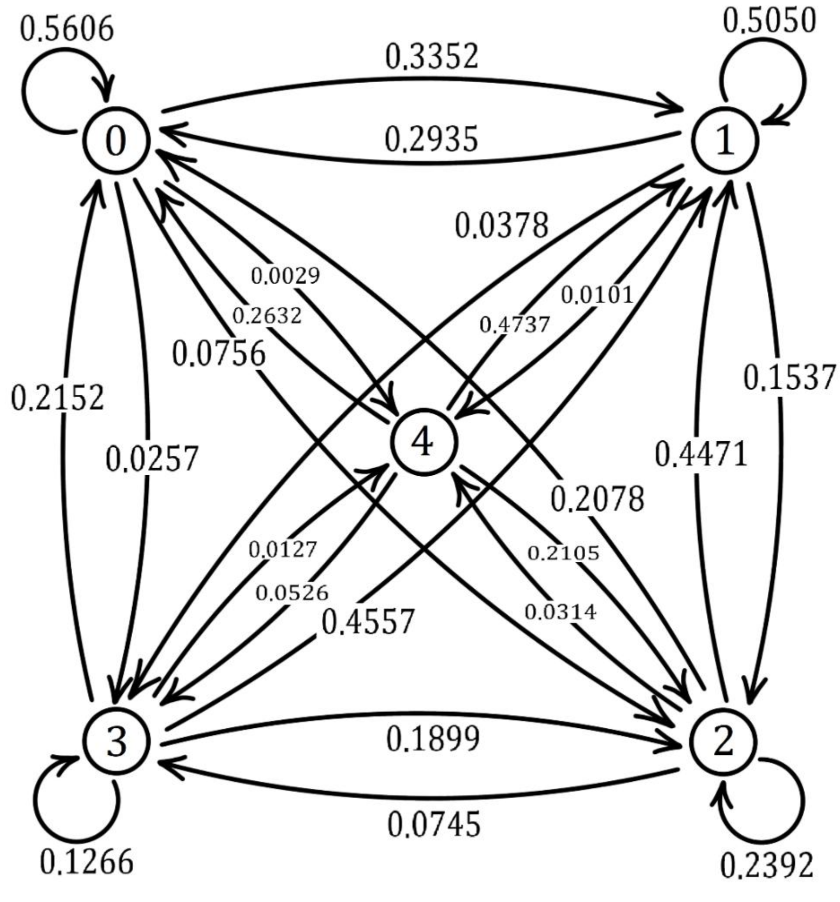

Figure 2. The Markov chain pattern of rainfall in Bengkulu City.

Based on the estimated Markov chain model, the following conclusions can be drawn for Bengkulu City. When the current day experiences no rainfall, the highest likelihood for the subsequent day is also no rainfall, with a probability of 56.06%. If the present condition is light rainfall, the most likely condition on the following day remains light rainfall, occurring with a probability of 50.50%. In the case of moderate rainfall, the greatest probability for the next day corresponds to light rainfall, at 44.71%. Similarly, when heavy rainfall occurs, light rainfall is the most probable condition on the next day, with a probability of 45.57%. Finally, if the current day is characterized by very heavy rainfall, the highest probability for the following day is again light rainfall, estimated at 47.37%.

3.4 Model Validation

To evaluate the performance of the proposed Markov Chain model, a validation process was conducted using an independent dataset from the year 2025 (testing data), consisting of 365 daily observations. The validation method compares the actual state of the next day (t + 1) with the state predicted by the maximum transition probability in Matrix P in the previous subsection.

Based on the transition matrix P derived from the training data (2020–2024), the maximum probability predictions are as follows:

If today is **No Rainfall**, the model predicts **No Rainfall** (P_{00} = 0.56).

If today is **Light, Moderate, Heavy, or Very Heavy Rain**, the model consistently predicts **Light Rain** for the next day (as P_{i1} is the dominant probability in rows 2, 3, 4, and 5).

The validation results against the 2025 testing data are presented in Table 6

Table 6. Validation Results using 2025 Data

Initial State (Day t)	Predicted State (Day t+1)	Total Observations	Correct Predictions	Accuracy (%)
No Rain	No Rain	156	91	58.33%
Light Rain	Light Rain	110	38	34.55%
Moderate Rain	Light Rain	55	16	29.09%
Heavy Rain	Light Rain	15	6	40.00%

Very Heavy Rain	Light Rain	28	11	39.28%
	TOTAL	364	162	44.51%

The overall prediction accuracy of the model is **44.51%**. While this value indicates moderate predictive power, it is significantly higher than a random guess probability for a 5-state model, which is 20%. Notably, the model performs best in predicting dry days (No Rain) with an accuracy of **58.33%**. This suggests that the Markov Chain model is particularly effective in capturing the persistence of dry spells in Bengkulu City, which is crucial for drought monitoring in the agricultural sector. However, the model tends to generalize rainfall events into "Light Rain" for the next day due to the dominance of the P_{i1} probabilities, which is a common characteristic of first-order Markov Chains in tropical regions where light rain is the most frequent transition.

To rigorously evaluate the performance of the proposed Markov Chain model, its accuracy (44.51%) and probability distribution were compared against more robust meteorological baselines rather than a simple uniform random guess (20%). The first baseline is the Persistence Model (assuming tomorrow's weather equals today's weather), which yielded an accuracy of 41.21% and a Macro F1-Score of 0.2796. The second baseline is the Majority Class Model (constantly predicting 'No Rain', the most frequent state in 2025), which yielded an accuracy of 43.13% but a very low Macro F1-Score of 0.1205.

While the Markov Chain model's accuracy (44.51%) shows a modest improvement over the deterministic baselines, the severe class imbalance in the rainfall data renders accuracy alone insufficient for evaluating stochastic models. The extremely low F1-Score of the Majority Class model (0.1205) demonstrates that relying solely on frequency leads to complete failure in predicting critical minority states (such as moderate to very heavy rain). Furthermore, the Markov Chain model achieved a **Brier Score of 0.7175**, outperforming the uniform random baseline (0.8000), proving its capability to capture dynamic transition probabilities rather than just deterministic point predictions.

3.5 Steady State Condition

The steady-state probability values for each state are obtained once the system reaches equilibrium. These steady-state probabilities can be achieved after the process has proceeded through n steps. Based on the matrix multiplications performed over n steps, it was found that at the 12th step, denoted as $P^{(12)}$, the system had reached its steady state. The matrix $P^{(12)}$ is obtained by raising the transition matrix P to the 12th power. From $P^{(12)}$, the steady-state probabilities π for each state, namely π_0, π_1, π_2, π_3 and π_4 are obtained. Another way of finding the steady state condition is by solving a linear equations system (as mentioned in Subsection 2.2) as follows.

$$\pi_0 + \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$$

$$\begin{aligned}
\pi_0 &= 0.5606\pi_0 + 0.2935\pi_1 + 0.2078\pi_2 + 0.2152\pi_3 + 0.2632\pi_4 \\
\pi_1 &= 0.3352\pi_0 + 0.5050\pi_1 + 0.4471\pi_2 + 0.4557\pi_3 + 0.4737\pi_4 \\
\pi_2 &= 0.0756\pi_0 + 0.1537\pi_1 + 0.2392\pi_2 + 0.1899\pi_3 + 0.2105\pi_4 \\
\pi_3 &= 0.0257\pi_0 + 0.0378\pi_1 + 0.0745\pi_2 + 0.1266\pi_3 + 0.0127\pi_4 \\
\pi_4 &= 0.0029\pi_0 + 0.0101\pi_1 + 0.0314\pi_2 + 0.0127\pi_3 + 0.0000\pi_4
\end{aligned}$$

The solution is

$$\begin{bmatrix}\pi_0\\ \pi_1\\ \pi_2\\ \pi_3\\ \pi_4\end{bmatrix} = \begin{bmatrix}0.380\\ 0.430\\ 0.138\\ 0.042\\ 0.010\end{bmatrix}$$

When the n-step transition probabilities have reached the steady state, it indicates that the Markov process has attained equilibrium or stability, meaning that the probabilities at subsequent steps no longer change. These steady-state probabilities can be used to predict the long-term probability of daily rainfall intensity in Bengkulu City.

Thus, it can be concluded that regardless of today's rainfall condition, the weather in Bengkulu City has the following probabilities: 38% for no rain, 43% for light rain, 13.8% for moderate rain, 4.2% for heavy rain, and 1% for very heavy rain.

4. Conclusion

Based on the Markov Chain model obtained, it can be concluded that in Bengkulu City: if today's weather condition is no rain, then the most probable weather condition for tomorrow is also no rain with a probability of 56.06%; if today's weather condition is light rain, then the most probable weather condition for tomorrow is also light rain with a probability of 50.50%; if today's weather condition is moderate rain, then the most probable weather condition for tomorrow is light rain with a probability of 44.71%; if today's weather condition is heavy rain, then the most probable weather condition for tomorrow is light rain with a probability of 45.57%; and if today's weather condition is very heavy rain, then the most probable weather condition for tomorrow is light rain with a probability of 47.37%.

Based on the obtained steady-state probabilities, it can be concluded that regardless of today's rainfall condition in Bengkulu City, the long-term probabilities for the weather are as follows: 38% for no rain, 43% for light rain, 13.8% for moderate rain, 4.2% for heavy rain, and 1% for very heavy rain.

This study has several methodological limitations that should be acknowledged. First, the application of an annual aggregate model assumes time homogeneity, which does not fully account for specific seasonal fluctuations throughout the year. Second, the first-order Markov Chain structure limits the prediction memory to only the preceding day's state, making it less optimal for capturing cumulative phenomena, such as prolonged periods of extreme rainfall.

As methodological recommendations for further development, it is suggested to adopt Non-Homogeneous Markov Chains by constructing seasonal transition matrices partitioned by temporal quarters or specific climatic seasons. Furthermore, exploration of Higher-Order Markov Models or Hidden Markov Models (HMM) is highly recommended to extract more complex temporal dependency structures. Finally, integrating spatial components into the transition chains could be considered to produce a more comprehensive prediction of weather patterns across neighboring regions

For future research, it is recommended that the analysis of rainfall patterns in a particular region should take into account other factors that may influence weather variations in Bengkulu City. Once more recent empirical data become available, it is also essential to validate the predictions generated by this Markov Chain model to ensure the model's accuracy and reliability.

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6. References

Arshinta, U. F., & Ahmad, D. (2019). Analisis Curah Hujan di Kota Padang dengan Menggunakan Rantai Markov. *Journal of Mathematics UNP*, *4*(4). https://doi.org/10.24036/UNPJOMATH.V4I4.7928

Data Online - Direktorat Data dan Komputasi BMKG. (n.d.). Retrieved March 7, 2026, from https://dataonline.bmkg.go.id/dataonline-home

Dunkerley, D. (2023). Recording Rainfall Intensity: Has an Optimum Method Been Found? *Water 2023, Vol. 15, 15*(19). https://doi.org/10.3390/w15193383

Elkenawy, E. S. M., Alhussan, A. A., Eid, M. M., & Ibrahim, A. (2024). Rainfall classification and forecasting based on a novel voting adaptive dynamic optimization algorithm. *Frontiers in Environmental Science*, *12*, 1417664. https://doi.org/10.3389/fenvs.2024.1417664

Hillier, F. S., & Lieberman Operations, G. J. (2010). *Introduction to Ninth Edition INTRODUCTION TO OPERATIONS RESEARCH*. www.mhhe.com

Karlin, S., & Taylor, H. E. (2012). A First Course in Stochastic Processes: Second Edition. *A First Course in Stochastic Processes: Second Edition*, 1–557. https://doi.org/10.1016/C2009-1-28569-8

Maharani, S., Ratnawati, H. I., & Lubis, A. M. (2025). Hubungan ENSO-IOD terhadap Curah Hujan dan Suhu Permukaan Laut di Perairan Bengkulu. *Buletin Oseanografi Marina*, *14*(2), 284–298. https://doi.org/10.14710/buloma.v14i2.69693

Pinsky, M. A. ., Karlin, Samuel., & Taylor, H. M. . (2011). *An introduction to stochastic modeling*. 563. https://books.google.com/books/about/An_Introduction_to_Stochastic_Modeling.html?id=RL74jxXd-zQC

Putra, M., Rosid, M. S., & Handoko, D. (2024). High-Resolution Rainfall Estimation Using Ensemble Learning Techniques and Multisensor Data Integration. *Sensors 2024, Vol. 24, 24*(15). https://doi.org/10.3390/s24155030

Nasib, S. K., Dicky Yanuari, E. D., & Machmud, T. (2022). Karakteristik Rantai Markov pada Data Curah Hujan Bulanan Stasiun Djalaluddin. *JMPM: Jurnal Matematika Dan Pendidikan Matematika*, *7*(2), 81–89. https://doi.org/10.26594/jmpm.v7i2.2654

Rumeon, S. G., Aulya, N., Telussa, S. W., Patty, C. A., Sopaliu, F. F., Rumalean, F., Rumangun, C. T., Tuankotta, W., & Yudistira. (2025). APPLICATION OF MARKOV CHAIN IN MONTHLY RAINFALL PREDICTION IN AMBON CITY. *Jurnal Statistika Dan Aplikasinya*, *9*(2), 36–45. https://doi.org/10.21009/jsa.09204

Sabrina, V., Adib Azka, M., Aditya Sugianto, P., Stasiun Meteorologi Gamar Malamo Galela, B., Utara, H., Stasiun Meteorologi Sultan Babullah, B., & Stasiun Meteorologi Ranai, B. (2021). KAJIAN METEOROLOGIS SAAT KEJADIAN BENCANA HIDROMETEOROLOGIS DI MALUKU UTARA. *Jurnal Widya Climago*, *3*(2), 53. https://ejournal-pusdiklat.bmkg.go.id/index.php/climago/article/view/35

Sanusi, W. (2024). Estimasi Bayes Empirik pada Model Rantai Markov untuk Menggambarkan Karakteristik Curah Hujan di Kota Makassar. *Journal of Mathematics: Theory and Applications*, *6*(1), 67–74. https://doi.org/10.31605/jomta.v6i1.3594

Tsani, N. M., Permana, D., Kurniawati, Y., & Salma, A. (2024). Markov Chain Application to Daily Rainfall Data in Semarang City. *UNP Journal of Statistics and Data Science*, *2*(3), 304–309. https://doi.org/10.24036/ujsds/vol2-iss3/189