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Unpacking the Complexity: Exploring Indonesian Secondary Students' Ways of Understanding Ill-Structured Math Problems

Shelly Morin[1], Didi Suryadi[2], Sufyani Prabawanto[3], Al Jupri[4]
[1,2,3,4]Pendidikan Matematika FPMIPA Universitas Pendidikan Indonesia
e-mail: *sufyani@upi.edu

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Abstract
The state space problem in problem solving involves a series of initial conditions, goal/end conditions, obstacles where this aspect is crucial in learning mathematics. This research aims this study aims to empirically investigate how students comprehend and navigate ill-structured mathematical problems. This research used a qualitative approach with case studies. Participants consisted of 12 secondary school students. Data were collected through observations, interviews, and document analysis of students' work. Data analysis was conducted thematically to identify patterns and key themes in how students solved unstructured problems. The findings showed that students faced difficulties in identifying relevant information, formulating a solution plan, and evaluating solutions in unstructured problems. The problem-solving strategies students used varied, ranging from heuristic approaches to the use of more formal mathematical models. In addition, factors such as self-confidence, prior experience, and support from teachers also play an important role in the problem-solving process. The implications of these findings include the need for an in- depth understanding of how students think and act in this context can provide valuable insights for the development of more effective teaching strategies, aimed at improving students' ability to deal with complex and ambiguous mathematical challenges.

Keywords: Ill-Structured problems, Mathematical Problem Solving, Students' Perspectives, Ways of understanding

1. Introduction

Guershon Harel offers a framework to analyze and understand the cognitive processes involved in mathematical problem solving. Specifically, this activity refers not only to the final answer that students produce but to the particular interpretation, solution, or explanation that students construct during the problem solving process (Harel, 2008). This cognitive product is referred to as a way of understanding. Ways of understanding, in other words, represent how students think about a mathematical concept, how they approach a problem, and how they justify their solutions. It is the product of students' mental actions, including problem interpretation, solution, and justification of mathematical statements.

According to Harel (2008), way of understanding what is usually called WoU, not only does WoU identify as a cognitive product but it also underlines the importance of principles that guide the development of effective WoU. These principles are known as the DNR principles, which stand for Duality Principle, Necessity Principle, and Repeated Reasoning Principle. Specifically, (1) the Duality Principle emphasizes that mathematical understanding often involves a reciprocal relationship between concepts and representations. Students need to be able to switch flexibly between various representations, such as symbols, graphs, and verbal language, to develop a comprehensive understanding (Panjaitan, 2019), (2) Necessity Principle argues that students tend to develop a deep understanding of mathematics when they feel the need to understand the concept to solve a problem or achieve a specific goal. Internal motivation and the need to understand play an essential role in the learning process (Kuba, 2023), (3) Repeated Reasoning Principle highlights the importance of providing opportunities for students to engage in repeated reasoning. This means students need to be given opportunities to solve similar problems, identify patterns, and formulate generalizations. Through repeated reasoning, students can build a stronger understanding and develop the ability to solve more complex problems (Zulmaulida, 2004).

Problem solving is at the heart of effective mathematics learning, serving as a bridge that connects abstract concepts with real-world applications (Harel & Sowder, 2013). In this context, it is important to distinguish between two main categories of mathematical problems: well-structured problems and ill-structured problems. Structured problems usually have a clear definition, complete information, and a single solution. In contrast, unstructured problems are characterized by ambiguous definitions, incomplete information, and often, multiple or uncertain solutions (Jonassen, 2010). Some previous studies are reported on table 1.

Table 1 *Previous research on understanding and ill-structured math problems*

No	Title	Reference
1.	Exploring The Critical Thinking Process of Prospective Teachers With High Mathematics Ability in Solving Ill-Structured Problems	Jaelani, dkk 2022
2.	Comparing Students' Problem-Solving Processes on Probability Tasks: Well-Structured and Ill-Structured Tasks 3	Auni & Kohar, 2023
3.	Exploring Mathematical Representations in Solving Ill-Structured Problems: The Case of Quadratic Function	Ika, dkk 2019
4.	The Influence of Students' Problem-Solving Understanding and Results of Students' Mathematics Learning	Sinaga dkk, 2023

5.	Students' Ways of Understanding and Ways of Thinking in Solving Trigonometric Problems	Magfiroh dkk 2024

Most previous research in the table 1 has examined ill-structured problems through the lens of specific skills such as critical thinking (Jaelani et al., 2022) or through direct comparison with well-structured problems that have a single solution and complete information (Auni & Kohar, 2023). Other approaches have focused on cognitive output, such as analyzing the mathematical representations students produce in solving problems (Ika et al., 2019). While these studies have made significant contributions, they tend to overlook a more fundamental aspect: how students actively construct meaning and mental structures when interacting with the vagueness of information, goal ambiguity, and lack of definitive solution procedures that characterize ill-structured problems.

This research identifies a crucial gap in the literature regarding the types of problems studied. Recent publications, such as those by Sinaga et al. (2023) and Magfiroh et al. (2024), while addressing problem solving, often do so in a general sense or in a specific domain such as trigonometry, without explicitly classifying the problems encountered as ill-structured problems. It is important to note that many "trigonometry problems" presented in class are well-structured problems. In contrast, this research explicitly targets ill-structured problems, which demand distinct cognitive processes. Ill-structured problems are characterized by incomplete or ambiguous information, the absence of a single correct solution, and the need for strong justification for each step in the solution.

According to Jonassen, ill-structured problems are characterized by several key elements. First, these problems do not have a single definite solution, but instead offer a variety of possible answers (Abdillah & Mastuti, 2018). Second, the information needed to solve the problem is often incomplete, ambiguous, or even contradictory. Third, the goal of the problem is also often unclear or must be defined by the problem solver. In addition, ill-structured problems tend to involve many variables and often require complex context considerations (Nurjanah, et al 2019). This is in contrast to well-structured problems, which are more well-defined, have clear solutions, and involve complete information.

Understanding ill-structured problems has essential implications in teaching and learning practices. Teachers need to design activities that encourage students to tackle complex and unstructured problems, rather than focusing solely on those with a single answer. A deeper understanding of ill-structured problem-solving holds significant implications for modern teaching and learning practices. This understanding underscores the need for educators to design learning activities that move beyond conventional, well-structured problems and instead, actively engage students with complex, ambiguous challenges. Such an approach is fundamental to developing essential skills like critical thinking, adaptive problem-solving, and the ability to apply knowledge across diverse contexts. While strategies like project-based learning and case studies have proven effective for introducing students to ill-structured problem-solving, a more granular analysis of their cognitive processes is required. Therefore, this study aims to empirically investigate how students comprehend and navigate ill-structured mathematical problems. Our primary focus is to identify the cognitive strategies they employ, the mental representations they construct, and the specific challenges they encounter throughout the problem-solving process.

2. Methods

This research employed a qualitative case study approach, focusing on students in class VIII of a junior high school To ensure a comprehensive and credible analysis, data were collected using three distinct methods: administering ill-structured geometry problems, classroom observations, and in-depth interviews. This multi-method approach, often referred to as triangulation, allows researchers to corroborate findings across different data sources, thereby strengthening the study's conclusions (Denzin & Lincoln, 2018). Following the problem-solving task, in-depth interviews were conducted to further explore the nuances of each student's solution process and overall understanding.

3. Result and Discussion

Students were given ill-structured mathematics problems, as in Figure 1. The problems that have been solved by students are then examined through interviews, which trace students' knowledge of understanding the problem and how they approach the problems they have solved. This does not simply focus on the correct final answer, but rather on the cognitive processes that underlie students' understanding of concepts, problem-solving strategies, and justification for the solutions they provide.

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Figure 1 *Ill-structured mathematics problem*

This problem contains unstructured math problems because: (1) Incomplete information i.e.

Perhatikan ilustrasi di bawah ini.

Kamu dan keluarga sedang dalam perjalanan mudik Lebaran. Setelah mengisi bensin di SPBU, ayahmu bertanya kepada karyawan SPBU mengenai jarak tempuh ke kota A dan kapasitas tangki mobil. Karyawan SPBU menjawab bahwa jarak dari SPBU ke kota A adalah sekitar 300 km dan menambahkan bahwa apakah kamu perlu mengisi bensin kembali tergantung pada kecepatan mobil.

Tangki mobilmu memuat 45 liter bensin. Konsumsi bahan bakar mobilmu dalam liter per kilometer tidak diketahui. Pertanyaan yang muncul adalah:

1. Apa faktor-faktor yang perlu dipertimbangkan untuk menentukan apakah kamu perlu berhenti untuk mengisi bensin lagi?
2. Jika asumsi konsumsi bahan bakar mobil adalah 1 liter per 12 km, apakah kamu perlu berhenti untuk mengisi bensin kembali sebelum mencapai kota A?
3. Bagaimana kecepatan mobil mempengaruhi konsumsi bahan bakar dan apakah ada kemungkinan bahwa perubahan kecepatan bisa mempengaruhi kebutuhan untuk berhenti mengisi bensin?

information is not given (car fuel consumption per kilometer) and there are assumptions that need to be made, (2) Reliance on real world context related to everyday situations (homecoming trips) involving real world knowledge, (3) These problems diverge from traditional formats by lacking a singular solution path or a universally correct answer. Answers can vary depending on the assumptions and considerations made. The elements that exist in unstructured math problems make students build their understanding of mathematical concepts in various situations tailored to each student.

Based on the results of students' answers to the first question, “What factors need to be considered to determine whether you need to stop filling the gasoline again?”, students have difficulty understanding the availability of gasoline in the car. Even though the question has given information, “you need to refuel depending on the speed of the car”. Although the problem provided an important clue, namely "you need to refuel depending on the speed of the car," many students failed to relate this information to the need to take fuel levels into account. This difficulty indicates students' lack of understanding of the causal relationship between speed, fuel consumption, and tank capacity. This limitation can be explained by several factors. First, students may have difficulty in applying theoretical knowledge about fuel consumption to a practical context. Second, a lack of hands-on experience with driving or managing personal vehicles may limit their understanding of the dynamics of fuel use. Third, question misinterpretation, which can be caused by a lack of ability to read and understand instructions carefully, also contributes to this difficulty.

According to research by Hijada & Cruz (2022), a student's reading comprehension level does not directly determine their success in mathematical problem-solving. This finding highlights that focusing on a single skill is insufficient. Therefore, designing effective interventions requires a holistic approach that considers the wide array of factors influencing a student's mathematical problem-solving abilities. After further research, Nahdi et al (2023) found a significant correlation between reading comprehension and mathematical problem solving ability. These results emphasize the importance of adopting a holistic perspective to mathematics education, integrating cognitive, affective, and literacy aspects. Integrating insights from these two perspectives can lead to a more nuanced understanding of the complexities of mathematics learning and provide a stronger basis for the development of effective educational interventions.

Based on the results of students' answers to the second question, “If the assumption of the car's fuel consumption is 1 liter per 12 km, do you need to stop to refuel before reaching City A?” Some students had difficulty in selecting and prioritizing crucial information from the question. They may fail to recognize the relationship between speed, distance travelled, and tank capacity, which is a key element in determining the right time to fill up. Many students tend to rely on heuristic approaches or rules of thumb, such as "I always fill up after 200 km." While heuristics can provide quick solutions, they often overlook important variables such as speed, road conditions, and the vehicle's fuel efficiency. Several students did not recheck their work after solving the problem. This can lead to unnecessary errors, such as simple miscalculations or misinterpreting information.

These three patterns of behavior collectively underscore the need for interventions that focus on enhancing conceptual understanding, developing more comprehensive problem-solving skills, and increasing metacognitive awareness in students. The use of simulations, more realistic case studies, and teaching strategies that encourage students to think critically and re-examine their work can help address these challenges. Other studies have shown that teaching strategies that emphasize conceptual understanding and the use of relevant examples can improve students' problem-solving skills (Chapman, 2015; Pertiwi et al 2022; Tóth, 2009).

Based on the results of students' answers to the first question, “How does the speed of the car affect fuel consumption, and is it possible that speed changes could affect the need to stop for gas?”. Students' difficulty in understanding questions containing the term "assumption" indicates significant challenges in academic literacy. Further analysis showed that students' unfamiliarity with academic terminology, such as "assumption," was a significant barrier. (1) Lack of experience with this term, compounded by (2) difficulty in understanding its meaning, triggered reluctance to answer the question. According to Stevens et al (2024), there is a correlation between increased word problem vocabulary and higher levels of word problem solving. Therefore, interventions that focus on improving academic vocabulary acquisition are crucial to improving students' performance in assessment contexts.

Analysis of students' responses to a series of questions related to vehicle refueling needs highlighted multidimensional challenges in problem solving and academic literacy. First, students struggled to relate theoretical concepts (speed, fuel consumption, tank capacity) to practical situations, possibly due to a lack of hands-on experience and difficulty in understanding the question instructions. Second, students tend to use a heuristic approach that simplifies the problem, ignores important variables, and potentially results in an inaccurate solution. Third, a lack of metacognitive awareness causes students to fail to double-check their work, increasing the likelihood of errors.

Another critical point also underlines the importance of a holistic approach in mathematics education, which considers cognitive, affective, and literacy aspects. The integration of conflicting research on the role of reading comprehension in math problem solving reinforces the need for comprehensive interventions. The improved acquisition of academic vocabulary, as evidenced by the difficulty in understanding the term "assumption," is also crucial.

4. Conclusion

This study concludes with several key findings regarding junior high school students' engagement with ill-structured mathematics problems. First, students commonly face difficulties in three critical areas: (1) identifying relevant information from the problem's context, (2) formulating a coherent solution plan, and (3) evaluating the reasonableness of their final solutions. Second, the problem-solving strategies employed by students are varied, ranging from informal heuristic approaches (such as trial and error) to the application of more formal mathematical models. Third, non-cognitive factors play a significant role; elements such as self-confidence, previous experience with similar tasks, and supportive teacher feedback are crucial in shaping the students' problem-solving process.

Based on these findings, this study offers several practical recommendations for mathematics educators seeking to enhance students' problem-solving skills: (1) Model "Think-Alouds": Teachers should explicitly model their own thinking processes when demonstrating how to solve ill-structured problems, making their strategies for identifying information and planning visible to students, (2) Teach Verification Techniques: Systematically instruct students on how to cross-check their work and evaluate the logic of their solutions, rather than just checking for calculation errors, (3) Introduce Heuristic Strategies: Formally introduce and practice various heuristic strategies (e.g., simplifying the problem, looking for a pattern, working backwards) as valid tools for problem-solving, (4) Foster a Supportive Environment: Create a classroom culture that supports experimentation and treats errors as learning opportunities, which can help build the self-confidence needed to tackle ambiguous problems.
problems.

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