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# IMPLEMENTATION OF DYNAMIC PROGRAM IN DETERMINING THE OPTIMUM PATH IN THE DISTRIBUTION OF PERTAMINA CHEMICAL & AGROCHEMICAL LUBRICATING OIL WITH FORWARD RECURSIVE EQUATION PT.POLA RAYA JAYA SAKTI

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**Abstract**
Determining the optimal distribution route is one of the crucial aspects in improving operational efficiency and reducing logistics costs, especially in the Chemical and Agrochemical Lubricating Oil industry. This study aims to apply a dynamic program as a calculation method to determine the optimum distribution route for PT Pola Raya Jaya Sakti, which acts as a Pertamina distributor. The method used is a forward recursive equation (stage 1 to stage n) that is able to analyze various possible distribution routes by considering variables such as distance, travel time, and vehicle capacity. Based on the research with the dynamic program with Forward Recursive Equation, A → C → D → E → H → I → K → L → N → O → Q → R →S. The total distance is A + C + E + H + I + L + N + O + Q + R = 0 + 8.5 km + 7.3 km + 4.4 km + 13 km + 70 km + 50 km + 16 km +9.5 km + 3 km = 181.7 km. The implementation of the dynamic program makes a real contribution in supporting a more effective and efficient distribution strategy, while strengthening the company's supply chain performance.

**Keywords:** Optimum Paths ,Dynamic Programs, Forward Recursive Equations

## 1. Introduction

Distribution is the activity of every entrepreneur or company in channeling, distributing, sending and delivering production to consumers. Distribution aims so that customers can receive production in good condition and on time. The distribution channel is the path that goods should pass from producers to wholesalers to consumers, also defined as the distribution channel of goods and services to reach consumers. With an efficient delivery channel, it will produce a good and efficient channel at a minimum cost (Maulana, 2022).

Dynamic programming is an algorithmic technique used to solve optimization problems by dividing them into smaller subproblems, solving the subproblems independently, and combining the solutions of the subproblems to obtain the optimal solution of the main problem (Thomas et al., 2022). Dynamic programming techniques decompose multistage decision problems as a sequence of single-stage decision problems (Asnawi N, 2020). The use of a dynamic programming approach to route determination provides several benefits such as being able to adapt delivery plans to changing conditions, such as changes in traffic, weather conditions, or changing customer demand. This allows companies to remain responsive and efficient in managing their supply chain. Overall, using a dynamic programming approach in routing allows companies to improve operational efficiency, reduce logistics costs, and increase customer satisfaction with faster and more reliable deliveries.

There are two dynamic program approaches, namely (Shania VA & Winarmo, 2022):

a. Forward recursive equation / up down is a calculation from front to back. In this case the dynamic program will move from stage 1 to stage n, while the decision variables are $x_1$,$x_{2,\ldots}$,$x_n$.

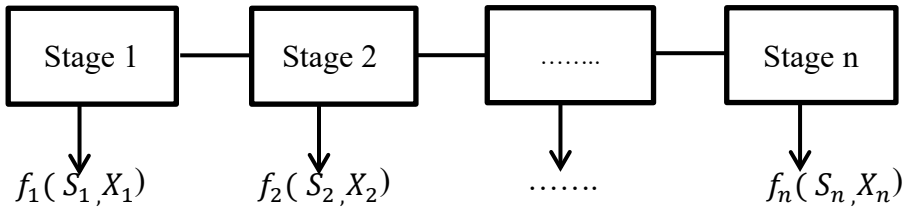


**Figure 1. Forward Dynamic Program**

*b.* Backward recursive equation/bottom up is a calculation that is done from back to front. In this case, the dynamic program will move from stage n, backwards to n-1, n-2 and so on until stage 1. The decision variables are $x_{n,}x_{n-1,\ldots,}x_{1,}$

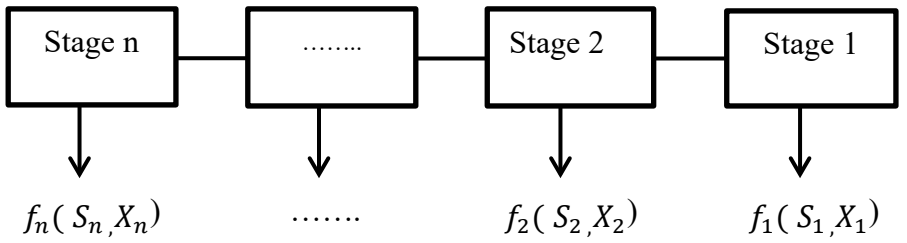


**Figure 2. Backward Dynamic Program**

The equation of recursion can be written as follows (Rina et al., 2024):

$$f_n(\mathrm{S}) = \mathrm{Min}\{(f_n(\mathrm{X_n}) + f_{n+1}(\mathrm{X_{n+1}})\} \quad (2.1)$$

Description

$f_n$ = Value at stage n

$X_n$ = Best route of stage n

S = Distance

Minimum Production Cost: $f_n\ (I_n)$

$f_n\ (I_n) = \text{Min}\ \{(AX_n + Y(I_n) + f_{n-1}(I_{n-1}))\}$ n=1,2,3,… (2.2)

Forward Recursive : $f_n\ (I_n) = \text{Min}\ \{(AX_n + B(I_n)) + f_{n-1}(I_n + s_n\text{-}X_n)\}$ (2.3)

Backward Recursive : $f_n\ (I_n) = \text{Min}\ \{(AX_n + B(I_n)) + f_{n+1}(I_n + s_n\text{-}X_n)\}$ (2.4)

## 2. Methods

Taking PT Pola Raya Jaya Sakti lubricating oil from the warehouse located at Jln.Sampali Road No.151, Kec.Percut Sei Tuan, Deli Serdang Regency and then distributed to PT Unilever Oleochemical Indonesia in Sei Mangkei, Kec. Bosar Maligas, Simalungun Regency, North Sumatra 21184. For determining km using the help of google maps. The variables in this study are the regional stages that must be taken at the stage.

**Table 1 Stages**

| *Stages* | Region |
|---|---|
| $X_1$ | Jln.Sampali Road No.151 |
| $X_2$ | Jln.Letda Sujono |
| $X_3$ | Jln. Panglima Denai |
| $X_4$ | Flyover Amplas |
| $X_5$ | Tanjung Morawa |
| $X_6$ | Tebing Tinggi |
| $X_7$ | Ring Road Lima Puluh |
| $X_8$ | Jln.Perintis Kemerdekaaan |
| $X_9$ | Simpang Kawasan Ekonomi Khusus KEK Sei Mangkei |
| $X_{10}$ | PT. Unilever Oleochemical Indonesia |

The procedures in this study are:

1. Collecting data from google map.

2. Selecting the variable Xn (n=1, 2, 3, ...) as the area that must be traveled in the stage.
3. Determining the Dynamic Programming recursive method that will be used. In this research, forward recursive is used.
4. From the calculation of the values at each stage, the optimal solution is obtained, namely the shortest distance.
5. Determine the path of the optimal result that has been obtained.

## 3. Result and Discussion

### 3.1 Data Analysis

The route used in this study is the Pertamina Chemical & Agrochemical Lubricating Oil distribution route of PT Pola Raya Jaya to PT Unilever Oleochemical Indonesia in Sei Mangkei. The travel routes are then made into a network that connects the intersections and locations that are passed on the way, so that the network diagram is obtained as follows:

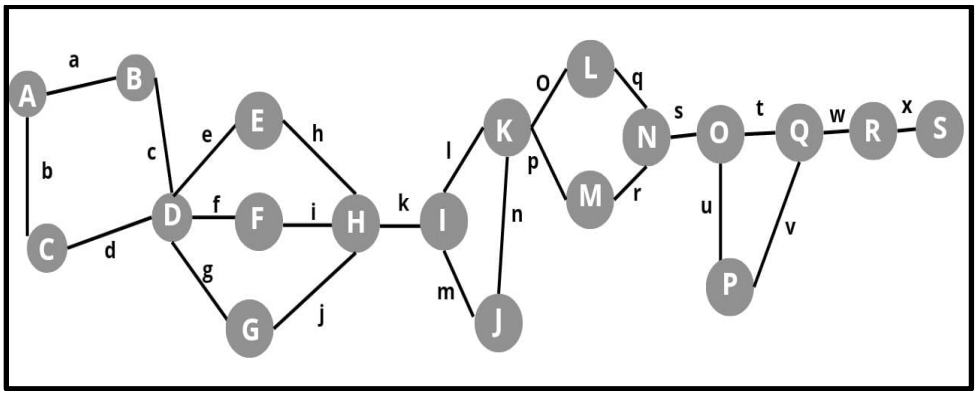


**Figure 3. Travel Route Sketch**

**Figure 4. Maps of PT.Pola Raya Jaya Journey to PT.Unilever Oleochemical Indonesia**

**Table 2. Code & Location**

| *Code* | *Location* |
|---|---|
| A | Jln. Sampali Road No 151 |
| B | Jln. Irian Barat Desa Sampali |
| C | Jln. Kesuma |
| D | Jln.Letda Sujono |
| E | Jln. Padang |
| F | Jln.Denai |
| G | Jln.Seksama |
| H | Jln.Panglima Denai |
| I | Flyover Amplas |
| J | Jln.Pertahanan |
| K | Tanjung Morawa |
| L | Lubuk Pakam |
| M | Galang |
| N | Tebing Tinggi |
| O | Ring Road Lima Puluh |
| P | Jln. Pelajar |
| Q | Jln.Perintis Kemerdekaaan |
| R | Simpang Kawasan Ekonomi Khusus KEK Sei Mangkei |
| S | PT. Unilever Oleochemical Indonesia |

**Table 3. Code & Distances**

| *Code* | *Distances* |
|---|---|
| a | 3.5 km |
| b | 3.6 km |
| c | 5.7 km |
| d | 4.9 km |
| e | 900 m = 0.9 km |
| f | 3.7 km |
| g | 10 km |
| h | 6.4 km |
| i | 3.7 km |
| j | 2 km |
| k | 4.4 km |
| l | 13 km |
| m | 850 m = 0.85 km |
| n | 13 km |
| o | 12 km |
| p | 28 km |
| q | 58 km |
| r | 44 km |
| s | 50 km |

| | |
|---|---|
| t | 16 km |
| u | 8 km |
| v | 9 km |
| w | 9.5 km |
| x | 3 km |

### 3.2 Dynamic Program Calculation with Forward Recursive Equation

**1. Stage I**

In stage 1 (n = 1) the trip starts from Jln. Sampali Road No 151 (A) to Jln. Sampali Road No 151 (A) = 0 km. The results can be seen in the following table:

**Table 4. Stage I**

| S | A | $F_1(x_1)$ | $x_1$ |
|---|---|---|---|
| A | 0 | 0 | A |

In the table above the feasible solution is Jln. Sampali Road No 151 (A)

**2. Stage II**

In stage 2 (n=2) the journey starts from B & C to D. The results can be seen in the following table:

**Table 5. Stage II**

| S | D | $F_2(x_2)$ | $x_2$ |
|---|---|---|---|
| B | 9.2 km | 9.2 km | C |
| C | 8.5 km | 8.5 km | |

$F_2$ = Min $F_{A,B}$ + $F_2$(D) = 3.5 km + 5.7 km = 9.2 km

$F_2$ = Min $F_{A,C}$ + $F_2$(D) = 3.6 km + 4.9 km = 8.5 km

In the table above the feasible solution is Jln. Kesuma (C)

**3. Stage III**

In stage 3 (n =3) the journey starts from E, F & G to H. The results can be seen in the following table:

**Table 6. Stage III**

| S | H | $F_3(x_3)$ | $x_3$ |
|---|---|---|---|
| E | 7.3 km | 7.3 km | E |
| F | 7.4 km | 7.4 km | |

| | | | |
|---|---|---|---|
| G | 12 km | 12 km | |

$F_3$ = Min $F_{D,E}$ + $F_3$(H) = 0.9 km + 6.4 km = 7.3 km

$F_3$ = Min $F_{D,F}$ + $F_3$(H) = 3.7 km + 3.7 km = 7.4 km

$F_3$ = Min $F_{D,G}$ + $F_3$(H) = 10 km + 2km = 12 km

In the table above the feasible solution is Jln. Padang (E)

**4. Stage IV**

In stage 4 (n = 4) the journey starts from H to I. The results can be seen in the following table:

**Table 7. Stage IV**

| S | I | $F_4(x_4)$ | $x_4$ |
|---|---|---|---|
| H | 4.4 km | 4.4 km | H |

$F_4$ = $F_4$(I)= 4.4 km

In the table above, the feasible solution is Jln. Panglima Denai (H)

**5. Stage V**

In stage 5 (n = 5) the journey starts from I & J to K. The results can be seen in the following table:

**Table 8 Stage V**

| S | K | $F_5(x_5)$ | $x_5$ |
|---|---|---|---|
| I | 13 km | 13 km | I |
| J | 13.85 km | 13.85 km | |

$F_5$ = $F_5$(K) = 13 km

$F_5$ = Min $F_{I,J}$ + $F_5$(K) = 0,85 km + 13 km = 13.85 km

In the table above the feasible solution is Flyover Amplas (I)

**6. Stage VI**

In stage 6 (n=6) the journey starts from L & M to N. The results can be seen in the following table:

**Table 9 Stage VI**

| S | N | $F_6(x_6)$ | $x_6$ |
|---|---|---|---|
| L | 70 km | 70 km | L |

| | | | |
|---|---|---|---|
| M | 72 km | 72 km | |

$F_6$ = Min $F_{K,L}$ + $F_6(N)$ = 12 km + 58 km = 70 km

$F_6$ = Min $F_{K,M}$ + $F_6(N)$ = 28 km + 44 km = 72 km

In the table above the feasible solution is Lubuk Pakam (L)

**7. Stage VII**

In stage 7 (n=7) the journey starts from N to O. The results can be seen in the following table:

**Table 10. Stage VII**

| S | O | $F_7(x_7)$ | $x_4$ |
|---|---|---|---|
| N | 50 km | 50 km | N |

$F_7$ = $F_7(O)$ = 50 km

In the table above, the feasible solution is Tebing Tinggi (N).

**8. Stage VIII**

In stage 8 (n=8) the journey starts from O & P to Q. The results can be seen in the following table:

**Table 11. Stage VIII**

| S | Q | $F_8(x_8)$ | $x_8$ |
|---|---|---|---|
| O | 16 km | 16 km | O |
| P | 17 km | 17 km | |

$F_8$ = $F_8(Q)$ = 16 km

$F_8$ = Min $F_{O,P}$ + $F_8(Q)$ = 8 km + 9 km = 17 km

In the table above, the feasible solution is Ring Road Lima Puluh (O)

**9. Stage IX**

In stage 9 (n=9) the journey starts from Q to R. The results can be seen in the following table:

**Table 12. Stage IX**

| S | R | $F_9(x_9)$ | $x_9$ |
|---|---|---|---|
| Q | 9.5 km | 9.5 km | Q |

$F_9$ = $F_9(R)$ = 9.5 km

In the table above the feasible solution is Jln.Perintis Kemerdekaan (Q)

**10. Stage X**

In stage 10 (n=10) the journey starts from R to S. The results can be seen in the following table:

**Table 13. Stage X**

| S | S | $F_{10}(x_{10})$ | $x_{10}$ |
|---|---|---|---|
| R | 3 km | 3 km | R |

$F_{10} = F_{10}(S) = 3$ km

In the table above, the feasible solution is Simpang Kawasan Ekonomi Khusus KEK Sei Mangkei (R).

Based on the results of the distance calculation, the route is obtained: A → C → D → E → H → I → K → L → N → O → Q → R →S, where the location sequence is Jln. Sampali Road No 151 → Jln. Kesuma → Jln.Letda Sujono → Jln. Padang → Panglima Denai → Flyover Amplas → Tanjung Morawa → Lubuk Pakam → Tebing Tinggi → Ring Road Lima Puluh → Jln.Perintis Kemerdekaaan → Kemerdekaaan Simpang Kawasan Ekonomi Khusus KEK Sei Mangkei → PT. Unilever Oleochemical Indonesia. The total distance is A+ C + E + H + I + L + N + O + Q + R = 0 + 8.5 km + 7.3 km + 4.4 km + 13 km + 70 km + 50 km + 16 km +9.5 km+ 3 km = 181.7 km. This information can be seen through the picture below .

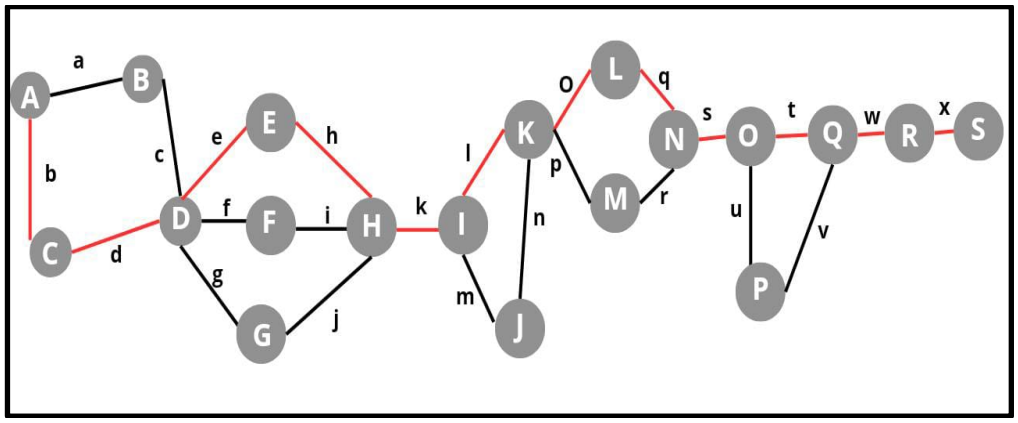


**Figure 5. Optimum Path Finding Results**

**4. Conclusion**

The application of the Dynamic Program with the Forward Recursive Equation method to the problem of distributing Pertamina Chemical & Agrochemical Lubricating Oil calculates the distance between each node on the path that has the minimum distance. The optimum path after

the dynamic program approach with the Forward Recursive Equation method is A → C → D → E → H → I → K → L → N → O → Q → R → S, where the location order is Jln. Sampali Road No 151 → Jln. Kesuma → Jln.Letda Sujono → Jln. Padang → Panglima Denai → Flyover Amplas → Tanjung Morawa → Lubuk Pakam → Tebing Tinggi → Ring Road Lima Puluh → Jln.Perintis Kemerdekaaan → Kemerdekaaan Simpang Kawasan Ekonomi Khusus KEK Sei Mangkei → PT. Unilever Oleochemical Indonesia. The total distance is A+ C + E + H + I + L + N + O + Q + R = 0 + 8.5 km + 7.3 km + 4.4 km + 13 km + 70 km + 50 km + 16 km +9.5 km+ 3 km = 181.7 km.

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