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Using Ant Colony Optimization to Solve a Vehicle Routing Problem: Waste Transportation Routes in Bengkulu City Case Study

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Abstract
The waste problem is one of the serious problems faced by every city in Indonesia, including Bengkulu City. To overcome the problem of waste production which continues to increase every day, the government's role is needed in efforts to transport waste with various facilities and support from the surrounding community. One effort to solve the problem of waste piling up in Temporary Disposal Sites (TPS) is to schedule effective transportation to Final Disposal Sites (TPA). To do the scheduling, one possible approach is a mathematical approach. The purpose of this research is to build a Vehicle Routing Problem (VRP) model using Ant Colony Optimization (ACO), and determine the solution of this model to determine the optimal solution to the problem of transporting waste from all TPS to TPA. This research involved 29 TPS in Bengkulu City. This model is solved using Python 2.7 programming language. With 2000 iterations, solutions were found in a relatively short time, which is less than 10 minutes. There are 6 garbage transportation routes in Bengkulu City which are considered as a VRP problem by using ACO which can transport garbage and considering the capacity of garbage trucks. From this route, a total distance of 128 km is obtained.

Keywords: Ant Colony Optimization, Vehicle Routing Problem, Waste Transportation Routes

1. Introduction

Waste refers to unused material that is discarded or removed from its original source, generated by living organisms and the surrounding environment, and holds no economic value. Waste can originate from daily activities such as households, industries, companies, offices, markets, hospitals, public facilities, and others. This waste is divided into organic and inorganic waste. If not properly managed, it can lead to the accumulation of waste, which may become a serious environmental issue. The increasing population inevitably impacts the volume of waste and environmental pollution.

The lack of awareness and concern from the community regarding waste issues has led to a decline in environmental quality. Therefore, government agencies play a crucial role in addressing this problem. Waste management is divided into two approaches: individual (local) and collective (centralized) patterns, which apply to residential areas or cities.

Waste management has become a significant issue, particularly in developing cities, one of which is the waste problem in Bengkulu City, which has yet to be properly addressed. Data from the Central Statistics Agency (Badan Pusat Statistik, BPS) in 2020 states that the population of Bengkulu City is 373,591 with an area of 151.70 km². According to data from the Environmental Agency (Dinas Lingkungan Hidup, DLH) of Bengkulu City, the amount of waste entering the final disposal site (Tempat Pembuangan Akhir, TPA) in Air Sebakul, Sukarami Village, Selebar District, Bengkulu City, can reach up to 250 tons per day. The large volume of waste generated daily is disproportionate to the capacity for waste management. The waste management process in Bengkulu City follows a collection, transport, and disposal system. The first step involves collecting waste in bins provided at the nearest Temporary Disposal Sites (Tempat Pembuangan Sementara, TPS), where the waste is usually sourced from households. Subsequently, garbage trucks from the DLH of Bengkulu City visit these collection points to load the waste into the trucks. Once the trucks are full, the waste is transported to the TPA at Air Sebakul.

The waste operational system is regulated by the Minister of Public Works of the Republic of Indonesia Regulation No. 3/PRT/M/2003. In its management, waste undergoes sorting, unification, transportation, processing, and final disposal. In the waste transportation process, vehicles such as garbage trucks are essential. The limited number of transport vehicles and infrastructure has resulted in waste management not being maximally handled. Currently, DLH owns 23 active waste collection vehicles, which are dumping trucks with a capacity of 8 km². These trucks are required to service 39 TPSs scattered across Bengkulu City, after which the waste is transported to the TPA in Air Sebakul.

To effectively utilize the available facilities, a strategy is needed in selecting optimal routes to ensure that waste can be transported from the TPSs to the Air Sebakul TPA quickly, without delays. Waste transportation is related to adequate transportation activities. One method that can be used to solve the route determination problem and minimize travel distance, considering the limited number of available vehicles, is the Vehicle Routing Problem (VRP) model. The VRP is an extension of the Travelling Salesman Problem (TSP). The basic VRP model assumes that the fleet is uniform, and each point must be serviced exactly once or more in one route, with the objective function being minimized, which is the travel distance that can be associated with costs.

In the process of transporting waste from the TPSs to the TPA, time and travel distance are crucial factors that must be considered, as they impact various aspects of operations, such as the number of vehicles needed, fuel costs, and the waste transportation routes. A common issue encountered in the waste transportation system is that the time required for each TPS may vary due to differences in the volume of waste generated at each TPS. Additionally, the limited number of available vehicles poses a challenge in waste transportation. Therefore, a systematic calculation is required to ensure that the routes taken by the garbage trucks from TPS to TPA are optimized, both in terms of the routes traveled and the associated costs. To the best of the author's knowledge, there has been no research addressing waste transportation in Bengkulu City using the VRP approach with ACO. By conducting this research, it is hoped that it will contribute to the development of knowledge and literature in this area.

The shortest route is the shortest path that will be traveled with a single destination point. Mutakhiroh et al. (2007) state that two methods can be used in finding the shortest route: conventional and heuristic methods. The conventional method is carried out through a basic mathematical estimation, while the heuristic method is performed with the aid of artificial intelligence to search and optimize solutions. The conventional method is relatively easier to understand compared to the heuristic method; however, the results obtained from the heuristic method are more diverse, and the solution time is shorter. The Ant Colony Optimization (ACO) algorithm is one of the heuristic methods also used to solve VRP cases.

ACO, the formula for which can be found in Section 2.2 of this paper, is inspired by the natural behavior of ant colonies, where ants travel from their nest to a food source, seeking the shortest route based on the trail left by ants along the path they take. As more ants traverse a particular route, the footpath trail becomes more pronounced. Thus, ACO can be applied to solve optimization problems related to determining the shortest distance by finding the optimal route to minimize time and costs.

There are several previous studies on the ACO method. Wulandari (2015) discussed how ACO can generate shorter routes compared to the initial route, resulting in improved time and cost efficiency during the waste transportation process. Another study was conducted by Afrianita (2011), which successfully addressed the VRP by minimizing the number of vehicles and reducing the total number of vehicles used. Soetomo (2018) was able to find the shortest path from one campus library to another.

2. Methods

This study applies the Vehicle Routing Problem (VRP) using Ant Colony Optimization (ACO) and determines the optimal solution for the waste transportation problem from all waste collection points (TPS) to the final disposal site (TPA). The data required for this study includes primary data obtained directly from the source through interviews with the Environmental Agency of Bengkulu City and field observations. Meanwhile, secondary data is sourced from outside the research area but remains relevant to the research topic, such as literature and distance data between locations obtained from Google Maps. The TPS selected for this study include the 30 largest TPS out of the 37 TPS in Bengkulu City.

The steps in analyzing the data are as follows:

1. Prepare the coordinate data of the TPS points to be visited.
2. Prepare the distance data between TPS points, where the data is based on the distance in kilometers.
3. Create a distance matrix between the TPS points.
4. Identify the parameter values for the ACO algorithm.
5. Generate random ant values.

6. Find the nearest point of probability based on the random values of the ants.
7. Create a new route to obtain the minimum distance route from the specified points.

2.1. The Vehicle Routing Problem (VRP)

The Vehicle Routing Problem (VRP) was first introduced by Dantzig and Ramzer (1959) in the context of determining routes and scheduling trucks. According to them, VRP is a problem related to the selection of vehicle routes, which holds significant importance in the industrial world, particularly in transportation and distribution management. Clarke and Wright (1964) later expanded on this study by introducing the concept of a depot as the point of departure and return for vehicles. The algorithm used by Clarke and Wright is the Saving Algorithm. VRP research remains active as it plays a crucial role in industries related to distribution, transportation, and logistics. The development of VRP has led to new analytical approaches and the emergence of new challenges.

The objective of the VRP is to distribute goods while minimizing costs through specific routes from the origin to multiple nodes (points) (Cordeau et al., 2007). The general goals of VRP, as stated by Toth and Vigo (2002), include:

1. Minimizing the total transportation cost related to distance and fixed costs associated with the vehicles.
2. Minimizing the total number of vehicles required to serve a number of clients.
3. Balancing the routes based on travel time and vehicle capacity.
4. Minimizing penalty costs resulting from poor service to clients.

According to Hermansyah (2011), the VRP is an extension of the Travelling Salesman Problem (TSP). The core issue in TSP is determining how a salesman should visit several cities, where the distances between cities are known, and each city must be visited exactly once. The significant difference between VRP and TSP lies in the fact that VRP involves the use of multiple vehicles to address the routing problem, while in TSP, a single salesman travels to visit cities to solve the problem.

Kurniawan et al. (2014) state that the VRP has several variations based on the number of constraints, such as time, distance, and the objectives to be achieved. Typically, these constraints involve minimizing duration, distance, and costs. The following are the variations found in VRP:

1. Time-Dependent VRP (TDVRP), which means the duration or cost of travel between two positions depends on the time of day.
2. Capacitated VRP (CVRP), where each vehicle has a load capacity limit.
3. Split Delivery VRP (SDVRP), where consumers are served by various vehicles.
4. VRP with Backhaul (VRPB), an extension of CVRP, where consumers are divided into two segments: linehaul customers (who receive deliveries) and backhaul customers (from whom items need to be picked up).
5. VRP with Time Windows (VRPTW), where each client must be supplied within a specific time window.

6. VRP with Pick-up and Delivery (VRPPD), an extension of CVRP where items must be picked up from a designated location and then delivered to a destination using the same vehicle.
7. VRP with Backhaul and Time Windows (VRPBTW), where customers are only served within a limited time frame.
8. VRP with Pick-up and Delivery and Time Windows (VRPPDTW), where items are picked up from one location and delivered to another using the same vehicle, with the additional constraint that each location has a specific service time interval.

2.2 Ant Colony Optimization (ACO)

Ant Colony Optimization (ACO) was first introduced by Dorigo et al. (1992) in their dissertation and was initially written and tested in 1996 under the name Ant System (AS). ACO is part of the swarm intelligence group, which belongs to a development paradigm used to find solutions to optimization problems, inspired by the behavior of insect swarms. According to Karjono et al. (2016), ACO is generally used to solve discrete optimization problems and complex cases with large variables.

ACO is an algorithm adapted from the behavior of ant colonies and how they interact with each other to detect the most efficient path from the nest to the food source and back. Dorigo et al. (1996) stated that during their journey to find food, ants leave pheromone trails along the paths they traverse. Pheromones, or foot tracks, are chemical compounds produced by endocrine glands that spread outside the body and can only be recognized and influence individuals of the same species. These pheromones play an important role in communication among ants, using the scent they produce.

The mechanism of the ACO algorithm includes the following steps:

1. Ants travel randomly.
2. When an ant encounters different paths, such as at a junction, it will randomly decide which direction to take.
3. Some ants will take the upper path, while others will take the lower path.
4. Upon finding food, the ants return to their nest while leaving a trail in the form of pheromones.
5. Since the path through the lower route is shorter, the ants traveling through that route will arrive faster, assuming all ants have the same speed.
6. The pheromones or foot tracks left on the shorter path will create a stronger scent compared to the longer path.
7. Other ants will be more likely to choose the lower path due to the more prominent pheromone scent.

In the ACO algorithm, variables and steps are required to find the shortest route, as outlined below:

Step 1:

a. Initialization of parameters used in ACO, including:

1. Pheromone intensity between points and its update (τ_{ij}).

2. The number of points (n), including the coordinates X and Y or the distance from one point to another (d_{ij}).
3. Determination of the origin and destination points.
4. Constant for the number of ant cycles or iterations (Q).
5. Constant controlling the pheromone intensity (α) with $\alpha > 0$.
6. Constant controlling the visibility (β) with $\beta > 0$.
7. Visibility between points $\eta_{ij} = 1/d_{ij}$.
8. Number of ants (m).
9. Pheromone evaporation constant (ρ) with $0 < \rho \leq 1$ to avoid infinite pheromone traces.
10. Maximum number of iterations (NC_{max}), which remains constant as the algorithm runs, while τ_{ij} will continuously be updated during each iteration of the algorithm from the first iteration ($NC = 1$) until the maximum iteration ($NC = NC_{max}$) or until convergence is achieved.

b. Initialization of the first point for the ants:
After the initialization of τ_{ij} is performed, m ants are placed at the first designated point.

Step 2:
Adding the first point to the tabu list, where the first result of initializing the starting point in the previous step must be loaded as the initial element of the tabu list. The outcome of this step is that the initial members of the tabu list for each ant are filled with a specific point marker so that each tabu list (l) can store the index of points from l to n.

Step 3:
Categorization of the paths taken by each ant to each point. The ant colony that has been allocated to the starting point will travel from the first point as the starting point, and one of the other points will be chosen as the destination. Then, from the second point, each ant colony will continue its journey by selecting one of the remaining points that are not included in the $tabu_k$ as the next destination point. The journey of the ant colony will continue until it reaches the final destination. If 's' defines the marker for the visit sequence, then the other starting points can be denoted by $tabu_k$ (s), and the other points are defined by $\{N - tabu_k\}$. To determine the final destination, the following probability equation is used to select the next point to visit:

$$p_{ij}^{k} = \begin{cases} \dfrac{[\tau_{ij}]^{\alpha} \cdot [\eta_{ij}]^{\beta}}{\sum_{k \in \{N - tabu_k\}} [\tau_{iki}]^{\alpha} \cdot [\eta_{ikj}]^{\beta}}, & j \in \{N - tabu_k\}, \\ 0, & \text{otherwise} \end{cases} \tag{2.1}$$

Where i denotes the starting point and j represents the destination city.

Step 4:
a. Calculating the route distance for each ant

The calculation of the closed route distance (L_k) for each ant is carried out after one iteration is completed by all ants. The calculation is made according to $tabu_k$ using the following Equation (2.2) as follows

$$L_k = d_{tabu_k(n),tabu_k(1)} + \sum_{k=1}^{n-1} d_{tabu_k(s),tabu_k(s+1)} \quad (2.2)$$

Where d_{ij} represents the distance between point i and point j, which is obtained from the following equation:

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (2.3)$$

b. Finding the Shortest Path
After all L_k of the ants have been calculated, the smallest value of the closed route length for each iteration, denoted as L_{minNC} and the smallest value of the overall closed route length, denoted as L_{min}ill be obtained.

c. Calculating the Change in Pheromone Intensity at Each Point
A group of ants will leave pheromone trails on the paths they traverse between points. There is exploration and variation in the number of ants passing through each path, which leads to a possibility of a change in the pheromone intensity at each point. The equation for this can be seen as follows:

$$\Delta\tau_{ij} = \sum_{k=1}^{m} \Delta\tau_{ij}^{k} \quad (2.4)$$

Where $\Delta\tau_{ij}^{k}$ represents the change in pheromone intensity between points for each ant, which can be calculated using the following equation:

$$\Delta\tau_{ij}^{k} = \frac{Q}{L_k} \quad (2.5)$$

Where i and j represent the origin and destination points in $tabu_k$, and

$$\Delta\tau_{ij}^{k} = 0 \quad (2.6)$$

for i, j otherwise

Step 5:
a. Calculating the pheromone intensity value between points for the next iteration.

The pheromone intensity value between all paths between points has the potential to change due to evaporation and the varying number of ants that pass through them. In the subsequent iterations, the ants will traverse these paths where the pheromone intensity has already changed. The pheromone intensity value between points for the next iteration is calculated using the following equation:

$$\tau_{ij} = \rho \cdot \tau_{ij} + \Delta\tau_{ij} \quad (2.7)$$

b. Re-setting the pheromone intensity change value between points.
In the following iterations, the change in pheromone intensity between points needs to be reset to zero to ensure proper calculation in the next iteration.

Step 6: Empty the tabu list and repeat Step 2 if necessary. The tabu list must be cleared to be populated again with a new sequence of points in the next iteration if the maximum iteration limit has not been reached or convergence has not been achieved. The algorithm needs to be repeated from Step 2 with the updated pheromone intensity values between points.

While ACO has proven to be an effective optimization technique for various combinatorial problems, it can sometimes take a considerable amount of time to converge to an optimal or near-optimal solution, particularly for large or complex problems. The algorithm often requires numerous iterations, which can be computationally expensive (Dorigo & Di Caro, 1999).

3. Result and Discussion

The data on the names and locations of the TPS that are the subject of this study, along with their positions (latitude, longitude), were obtained from Google Maps. 1° is equivalent to 111.319 km, and to generate unique values, the X and Y coordinates were obtained by multiplying each latitude and longitude by 111319. This data is presented in Table 1 as follows.

Table 1. TPS Data

TPS Number	TPS Name	Latitude	Longitude	X Axis	Y Axis
1	TPA Air Sebakul	-3,8246850	102,345335	-425,760109	11392,98035
2	TPS Lempuing	-3,8250617	102,278366	-425,802043	11385,52550
3	TPS AL	-3,8869819	102,315922	-432,694938	11389,70620
4	TPS Ps Minggu Atas	-3,7946962	102,266909	-422,421786	11384,25004
5	TPS Ps Minggu Bawah	-3,7927946	102,266837	-422,210102	11384,24212
6	TPS Simpang Jl Salak	-3,8150923	102,299874	-424,692259	11387,91974
7	TPS Ps Barukoto I	-3,7876946	102,250862	-421,642375	11382,46374
8	TPS Ps Barukoto II	-3,7890839	102,248684	-421,797030	11382,22129

9	TPS Berkas	-3,7996306	102,255546	-422,971078	11382,98514
10	TPS Lapas Malabero	-3,7909151	102,248228	-422,000878	11382,17049
11	TPS Pasar Bengkulu (Kota Tua)	-3,7728155	102,263334	-419,986048	11383,85217
12	TPS Terminal Panorama (LLAJ)	-3,8162849	102,299060	-424,825018	11387,82916
13	TPS Jln. Kedondong	-3,8170540	102,299952	-424,910634	11387,92843
14	TPS Belakang Balai Buntar	-3,8221380	102,294907	-425,476580	11387,36675
15	TPS RS M.Yunus	-3,8341200	102,313259	-426,810404	11389,40968
16	TPS SPBE Bumi Ayu	-3,8756797	102,338703	-431,436788	11392,24212
17	TPS Kampung Kelawi	-3,7817313	102,265631	-420,978546	11384,10780
18	TPS Tanggul Rawa Makmur	-3,7811510	102,279452	-420,913948	11385,64638
19	TPS Pantai Jakat	-3,7820745	102,261648	-421,016751	11383,66449
20	TPS Depan DPR (Pasar Pedati)	-3,7533096	102,304038	-417,814671	11388,38322
21	TPS Lap. Golf	-3,8419462	102,301563	-427,681609	11388,10774
22	TPS Polda	-3,8354560	102,309938	-426,959126	11389,03999
23	TPS Kebun Geran	-3,7947349	102,260527	-422,426094	11383,53969
24	TPS Kelautan dan Perikanan	-3,7855378	102,253936	-421,402282	11382,80595
25	TPS Belakang Stadion	-3,7929292	102,273431	-422,225085	11384,97613
26	TPS Kinibalu	-3,8023972	102,281861	-423,279053	11385,91454
27	TPS Anggut Atas	-3,7985508	102,261151	-422,850876	11383,60908
28	TPS Jembatan Ps Bengkulu	-3,7713152	102,264754	-419,819036	11384,01025
29	TPS Simpang Ps Bengkulu	-3,7668196	102,264896	-419,318591	11384,02605
30	TPS Kebun Beler	-3,8074622	102,271320	-423,842884	11384,74117

The number of vertices involved in this research is quite large, making manual computation highly ineffective. Therefore, in this study, the solution was sought with the assistance of software, specifically Python. The program was run on a computer with the specifications of an 11th Gen Intel(R) Core(TM) i7-1165G7 2.80 GHz processor and 16 GB of RAM. With 2000 iterations, the solution was obtained within a reasonable computation time of less than 10 minutes. In this study, 30 TPS were involved. For a larger number of TPS, the computation time is likely to increase. Further research is required on the topic of waste transportation in the future.

The output obtained from running the Python program is as follows:

Route 1:

TPS Pasar Bengkulu – TPS Jembatan Pasar Bengkulu – TPS Simpang Pasar Bengkulu – TPS Kelautan dan Perikanan – TPS Pasar Baru Koto II – TPA

Route 2:

TPS Pasar Minggu Bawah – TPS Pasar Minggu Atas – TPS Anggut Atas- TPS Berkas – TPA

Route 3:

TPS Kampung Kelawi – TPS Pantai Jakat – TPS Lapas Malabero- TPS Ps Barukoto I – TPS Kebun Geran – TPA

Route 4:

TPS SPBE Bumi Ayu – TPS AL – TPS Lapangan Golf – TPS Polda – TPS M Yunus – TPA

Route 5:

TPS Depan DPR – TPS Tanggul Rawa Makmur – TPS Kinibalu – TPS Kebun Beler – TPS Lempuing – TPA

Route 6:

TPS Pasar Panorama – TPS Jalan Kedondong – TPS Simpang Jalan Salak – TPS Belakang Balai Buntar – TPA

The total distance traveled by the six routes is 128 km

The following Figure 1 illustrates the waste transportation routes in Bengkulu City, considered as a VRP problem using ACO.

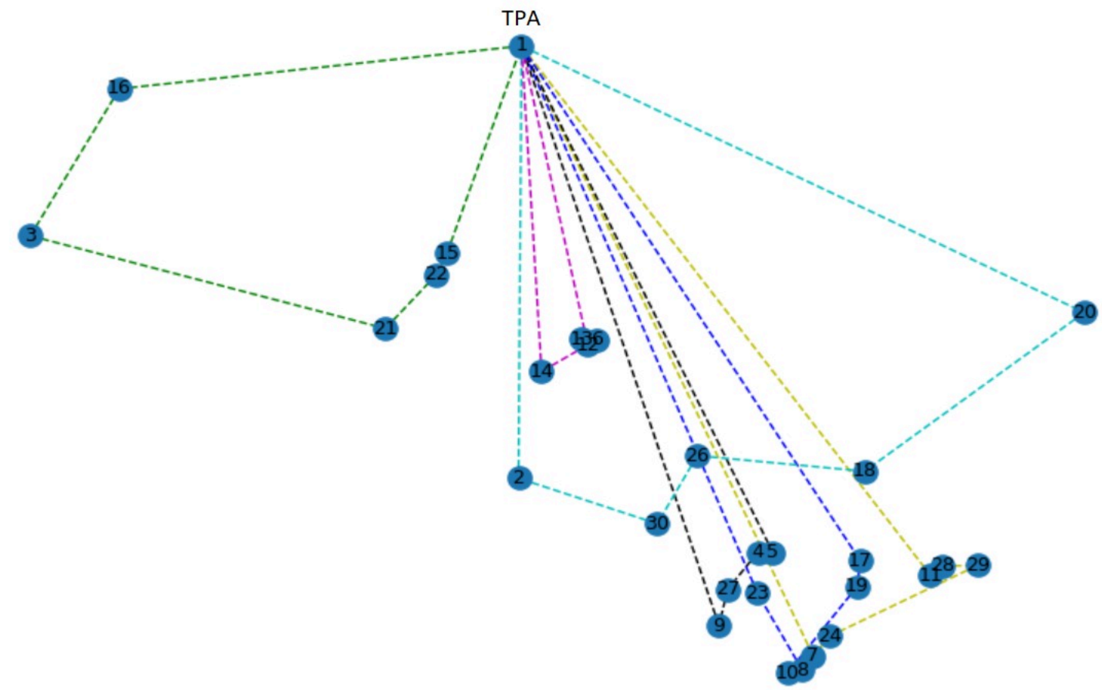

Figure 1. Waste Transportation Routes in Bengkulu City

The results obtained show 6 waste collection routes in the city of Bengkulu, viewed as a VRP problem using ACO, which can accommodate waste collection while considering the capacity of the waste collection trucks. From these routes, a total distance of 128 km was achieved. This research is expected to be beneficial as input for the Environmental Agency of Bengkulu City regarding the optimization of waste truck routes from the TPS to the TPA.

4. Conclusion

The Vehicle Routing Problem (VRP) with the Ant Colony Optimization (ACO) approach provides relatively good solutions with a relatively short computation time when the program is run on a computer with the specifications of an 11th Gen Intel(R) Core(TM) i7-1165G7 2.80 GHz processor and 16 GB of RAM. The results obtained show 6 waste collection routes in the city of Bengkulu, treated as a VRP problem using ACO, which can accommodate waste collection while considering the capacity of the waste collection trucks. From these routes, a total distance of 128 km was obtained.

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