EDUMATSAINS
JURNAL PENDIDIKAN MATEMATIKA DAN SAINS
E-ISSN:2527-7235
Volume 9 Issue. 2 January, 2025, pp 77-89
http://ejournal.uki.ac.id/index.php/edumatsains

Level of Students Geometry Thinking on quadrilateral topic

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Article Info
Article history:
Received : December 11, 2024
Revised : January 17, 2025
Accepted : January 21, 2025
Available online : January 31, 2025

https://doi.org/10.33541/edumatsains.v9i2.6411

Abstract

Quadrilateral is one of the geometry topics in the curriculum, and students at junior high school must understand the topic. Therefore, we researched 40 students (11/13 years) in Kambera District, East Sumba Regency, East Nusa Tenggara, to identify and describe students' geometric thinking levels. The study is a qualitative descriptive research that uses written tests as the data collection method, focusing on the quadrilateral case. The data were analyzed using van Hiele's theory. The results showed that 1) only one-third of the students are at level 0 (visualization), i.e., students knew geometric pictures by observing the images or models. At this level, half of the students still need to improve in classifying the types of quadrilaterals, and 2) only a few students are at level 1. They can understand or know the properties of quadrilaterals. The written results found that almost all students needed help understanding the topic of quadrilaterals and their properties well. Nearly all students have yet to be able to recognize and classify the types of quadrilaterals well.

Keywords: quadrilateral, geometry, van_Hiele

1. Introduction

Among the various mathematical topics, geometry touches almost all aspects of life. Many objects resemble geometric shapes like ventilation, windows, doors, and kites. (Schmitt, 2006) states the importance of geometry needs to be studied, "Geometry touches on every aspect of our lives." It is essential to examine the shapes, lines, angles, and spaces that are integrated into the daily lives of our students and ourselves.

Quadrilateral is one of the studies of geometry in school mathematics, where most students still need help, such as drawing rectangular shapes according to their type. Misconceptions often occur when understanding the concept of a quadrilateral. Clements and Battista (Nuraeni, 2010) revealed that students assume every shape with four sides is a square. (Agustyarini Y., 2023) In her research, she stated that there are misconceptions about understanding the characteristics of quadrilaterals, especially trapezoids and parallelograms. Students wrongly assume that the diagonals of an isosceles trapezoid are the only pair of line segments that are the same length. Students also assume that only parallelograms and rectangles are regular plane shapes. They do not consider variations in position or irregular shapes as characteristics of quadrilaterals. This is in line with (Budiarto, 2020), the first misconception experienced by students in the material on quadrilaterals lies in the understanding of quadrilaterals. Students' misconceptions in defining lie in plane shapes that have sides of the same length, four corners, and two diagonals forming right angles. This shows that students only choose regular quadrilaterals. In addition, students also experience misconceptions when distinguishing sides and edges. The second misunderstanding lies in the concept of the properties of rectangular plane shapes, primarily rectangles. Students assume that rectangles always have an elongated shape without paying attention to their properties. Next, there is a misunderstanding when showing other plane shapes. Students assume that stairs are slanted rectangles. In addition, if the rectangle has a different position or irregular shape, it is not called a rectangle.

The research results above show that students' geometry abilities are still relatively low. Mathematics' teaching and learning process affects students' difficulty in understanding the concept of geometry, i.e., students, teachers, facilities and infrastructure, and assessment (Nuraeni, 2010). An important aspect that influences the success of geometry teaching is the depth of subject matter knowledge possessed by current and prospective teachers (Chen, 2021). This expertise significantly determines student outcomes in geometry. Misconceptions about quadrilaterals are not only made by students but also by teachers. Soedjadi (Rifki, 2012) said teachers still have misconceptions about "length" when understanding rectangles. Elementary school teachers must attain deductive geometric thinking skills (Jupri, 2018). However, research indicates that many teachers and pre-service teachers have not reached the level of deductive reasoning (Decano, 2017; Denizli, 2018). Studies focusing on the geometric thinking levels of mathematics pre-service teachers show that most are at level 3 (Bulut, 2012; Fitriyani, 2018), while level 4 is scarce and challenging to attain. Students also often need help determining the rectangle depicted with a smaller size or rotated position. Teachers' lack of knowledge and skills in recognizing and developing students' abilities dramatically affects learning results. Limited resources and expertise in determining the level of student understanding of geometry and how to handle it is the cause of implementing conventional learning in schools. This is in line with (Niyukuri, 2020), who stated that some teachers skipped or postponed geometry teaching in Burundi due to inadequate knowledge of pedagogical content. The recurrence of this problem in various settings emphasizes the need for immediate intervention in geometry education.

One of the interventions teachers can do to solve students' misconceptions in geometry is to know the students' geometric thinking level. Therefore, teachers need media and processes to make them know the level of student understanding in geometry to develop learning as a solution to this problem. In addition, students also really understand their level of performance in order to develop and improve themselves. The van Hiele theory is one of the learning theories that can enhance students' thinking in geometry (Hock, 2015). It serves as a guide for teachers during instruction and as a tool to evaluate students' abilities. Utilizing the van Hiele theory is considered an effective approach to teaching geometry. Van Hiele's Geometry Theory is a learning theory proposed and developed by Pierre M. Van Hiele, a realistic mathematician and expert in geometry studies. Van Hiele argued that students' geometric thinking progresses in five sequential levels: level 0 (visualization), level 1 (analysis), level 2 (informal deduction), level 3 (deduction), and level 4 (rigor) (Usiskin, 1982; Clements. D. H., 1992; Walle, 2001). According to (Walle, 2001), high school students are typically at level 2 (informal deduction). Students learn geometry in several stages: introduction, analysis, classification, deduction, and precision.

The first level: Level 0, visualization. At this level, students can learn the names of shapes and identify the overall form (for example, distinguishing squares from rectangles). They recognize geometric shapes primarily by their visual features, viewing the object as a whole but needing to concentrate on the specific properties of the shape they observe. Therefore, at this level, students need help understanding and determining the geometric properties and characteristics of the shapes shown (Clements. D. H., 1992). For example, at this level, they know the door's shape as rectangular, but they have yet to realize it as a whole from the rectangular shape.

The second level: Level 1, analysis. At this level, students can identify the properties of shapes (e.g., rectangles have four right angles). They analyze the concepts and properties of geometric figures, using methods such as observation, measurement, drawing, and modeling to determine these properties. However, students have not been able to explain the relationship between these properties fully, have not been able to see the connection between several geometric shapes, and have not been able to understand the definition (Clements. D. H., 1992). For instance, students can identify a shape as a rectangle at this level because it has four sides and right angles.

The third level: Level 2, informal deduction. Students can logically arrange figures and understand relationships but must work within a formal mathematical system. While simple deductions can be made, a deeper understanding of proof is still needed. At this level, students can use informal deduction to recognize the relationship between the properties of a geometric figure and those of different shapes. They can classify shapes hierarchically. Students can also understand the connections between the properties of shapes at this stage of thinking. For example, in a parallelogram, the opposite sides are parallel, making the opposite angles equal, and a square is recognized as a rectangle because it shares all the rectangle's properties. Thus, students' reasoning enables them to form abstract definitions, offer informal arguments, and sort shapes according to their properties.

The fourth level: Level 3, Deduction. At this level, the student grasps the importance of deduction and understands the roles of postulates, theorems, and proofs. Students can write proofs with comprehension. At this level, students not only receive evidence but are already able to compile evidence. Students can make a list of axioms and definitions to create theorems. Students also prove the theorem using logical thinking, compared to thinking in stage 2, which tends to be more informal. (Usiskin, 1982) discovered that at this stage, students clearly understand the roles of concepts such as definitions, axioms, and theorems in geometry.

The fifth level: Level 4, Rigor. The student grasps the importance of rigor and can make abstract inferences (such as understanding non-Euclidean geometry). Students engage in formal reasoning within mathematical systems at this stage and can evaluate the implications of altering axioms and definitions. They can comprehend the connections between undefined terms, axioms, definitions, theorems, and formal proofs. (Clements. D. H., 1992) refers to this level of rigor as the "metamathematics level." At this stage, mathematicians reason formally within mathematical frameworks and analyze the effects of manipulating axioms and definitions. This level requires advanced and complex thinking, so high school students seldom reach it.

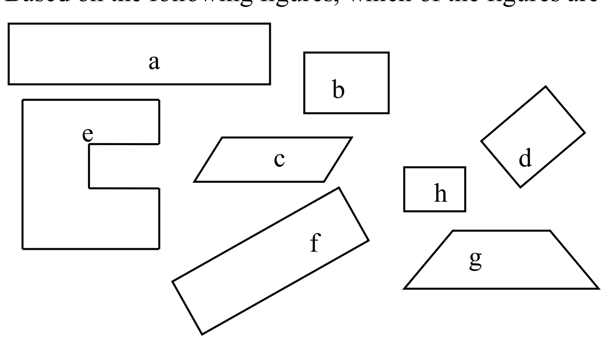
The geometric thinking levels in Van Hiele's theory have specific characteristics: (1) Students progress through the levels sequentially. As they move through a level, they engage in geometric thinking at that level and develop concepts that will serve as the foundation for the next level. (2) The thinking levels in Van Hiele's theory are not determined by age but are more influenced by the content, methods, and media used in learning, as well as the maturity of the students. Teachers must provide learning experiences that align with the student's cognitive stage. (3) The most significant factor in the speed at which students advance through the levels is their experience with geometry.

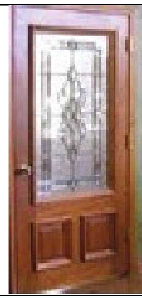
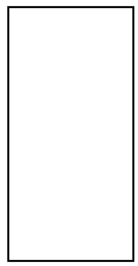
Van Hiele's theory discusses the stages or levels of students' understanding of geometry and how teachers can "align" these stages to achieve optimal learning outcomes (Van Hiele, 1999). Understanding students' thinking processes can maximize their intellectual potential. By analyzing these processes, we can determine how to enhance their thinking skills. Therefore, it is important to identify the Geometric Thinking Level of Junior High School students in East Sumba regarding Quadrilateral Geometry, according to Van Hiele's Geometry Theory.

2. Methods

This qualitative research aims to describe or explore a symptom, event, or incident thoroughly and deeply (Usiskin, 1982). Thus, the data obtained from this study were analyzed descriptively to determine the level of students' geometric thinking in solving flat mathematical problems, namely quadrilaterals. The data was gathered using an instrument tested on 30 students (11/13 years) from Kambera District, East Sumba Regency, East Nusa Tenggara. The students who were given the test were 8th-grade junior high school students. This study investigated students' mathematical geometric thinking levels related to quadrilateral material. This article presents the results for only the students with the highest and lowest test scores. The written test consists of four descriptive questions and is adjusted to the geometric thinking level. The written test is shown in Table 1. The students' geometric thinking levels were analyzed based on Van Hiele's theory, with the test structured around the characteristics of his framework, where each level reflects the thought process in geometry.

Table 1. The instrument of mathematical geometric thinking level

No	Mathematics Problem	Level
1	Based on the following figures, which of the figures are rectangles?  	Level 0 (Visualization)

<table>
<tr><th>No</th><th>Mathematics Problem</th><th>Level</th></tr>
<tr><td></td><td>
(Source: Journal for Research in Mathematics Education)</td><td></td></tr>
<tr><td>2</td><td>Fill in the table below according to the example!
<table>
<tr><th>No</th><th>Object</th><th>Sketch</th><th>The name of object</th></tr>
<tr><td>a</td><td></td><td></td><td>Rectangular</td></tr>
<tr><td>b</td><td></td><td>.....</td><td>.....</td></tr>
<tr><td>c</td><td></td><td>.....</td><td>.....</td></tr>
</table></td><td>Level 0
(Visualization)</td></tr>
<tr><td>3</td><td>A shape has the following characteristics.
- Has four sides that are the same length
- Has four right angles
- The diagonals are the same length.
What quadrilateral is that?
(Source: Journal for Research in Mathematics Education</td><td>Level 1
(Analysis)</td></tr>
<tr><td>4</td><td>Pay attention to the questions below.
Susi is discussing with Nita. Susi argues that a parallelogram is "a shape with four sides and the opposite sides are parallel". In contrast, Nita argues that "a parallelogram is a quadrilateral whose opposite sides are the same length". Who do you think is right? Can both be true? Explain your reasons!</td><td>Level 2
(Informal Deduction)</td></tr>
</table>

No	Mathematics Problem	Level
	(Source: The Van Hiele Model of the Development of Geometric Thought)	

3. Result and Discussion

This section will begin by discussing the overall findings and analyzing the students' geometric thinking levels in solving quadrilateral problems. A total of 30 students participated in the test.

Level 0 (Visualization)

At this level, students are expected to identify geometric shapes based on an object's visual characteristics. They perceive the object as a whole but must focus on its specific properties. Therefore, at this level, students need help understanding and determining the geometric properties and characteristics of the shapes shown (Clements & Battista, 1992). For example, at this level, they know the door's shape as rectangular, but they have yet to realize it as a whole from the rectangular shape. Geometry thinking level 0 (visualization) is found in questions number 1 and number 2. For question number 1, all students aged 11/13 answered that a rectangle is picture-a. Only one student aged 12 years didn't solve the problem. The answer given is correct, but the rectangle is not just picture-a. Other rectangles are also represented by picture-b, picture-f, and picture-h. Figure 1 shows the student's answer to question number 1. While in question number 2, students aged 13 years have not been able to know the shape of objects as a shape well. It can be seen in Figure 2. We can see that students mention the name of the kite object as "belah ketupat," which means rhombus. The best answer is shown in Figure 3. The student answers "layang-layang' means kite, and "persegi" means square.

Figure 1.

Example of student's answer based on questions number 1

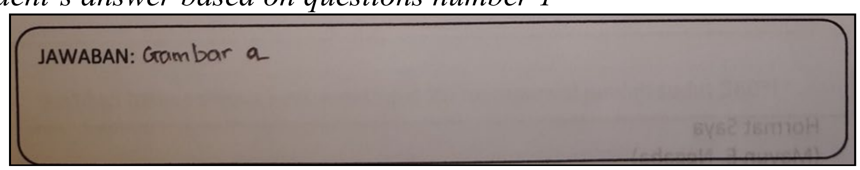


Figure 2.

Example of students (13 y.o) answer based on question number 2

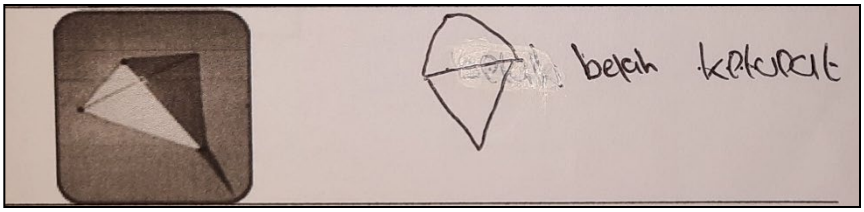
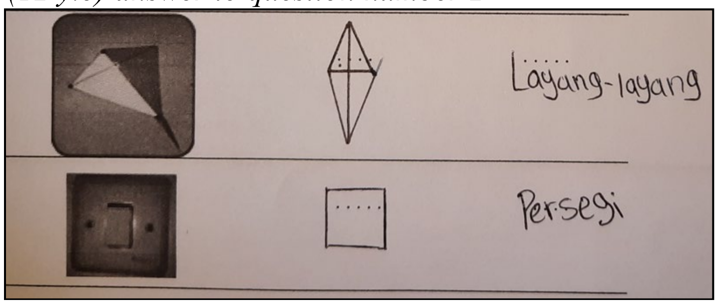


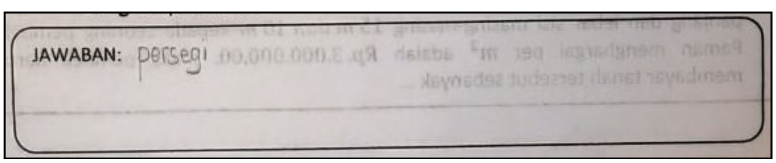
Figure 3.
Example of students' (12 y.o) answer to question number 2



Level 1 (Analysis)

At this level, students are expected to recognize the shape by analyzing the properties of the given figures. This indicates that students are engaged in analyzing the concepts and properties of geometric shapes. They can determine a shape's properties through methods such as observation, measurement, drawing, and modeling. Almost students give the correct answer (11/13 years old). According to the student's responses, it is seen that they can identify properties of shape. Students can say that the shape is a "purse," which means square because it has four sides, and all angles are right angles. Figure 4 shows the best answer for students.

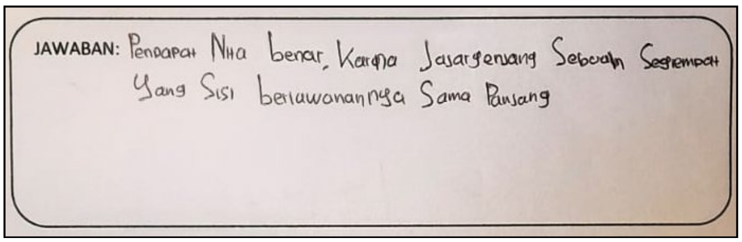
Figure 4.
Example of student's answer on question number 3. The student's answer is "persegi" means square



Level 2 (Informal Deduction)

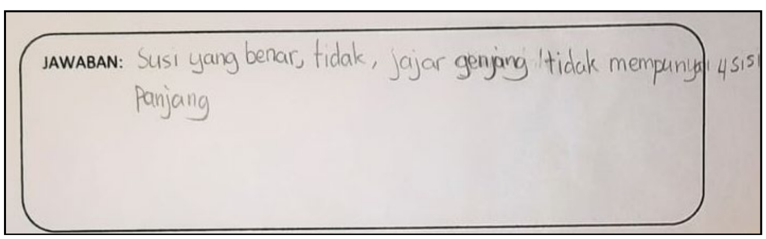
At this level, students are expected to recognize the relationship between the properties of a geometric figure and those of different shapes through informal deduction, and they are capable of classifying shapes hierarchically. Level 2 of geometric thinking is contained in question number 4 in Table 1, and all students need help to answer question number 4 correctly. For example, one student's answer: "pendapat Nita benar karena jajargenjang sebuah segiempat yang sisi berlawanannya sama panjang," means "Nita's opinion is correct because a parallelogram is a quadrilateral whose opposite sides are the same length" (Figure 5). Figure 6 shows the other example. The student's answer: "Susi yang benar, jajargenjang tidak mempunyai 4 sisi," means "Susi is correct; a parallelogram does not have 4 sides". The correct answer for problem number 4 is "both opinions are correct because, according to the definition, a parallelogram is a quadrilateral whose two pairs of opposite sides are parallel and have the same length."

Figure 5.
Example of student's answer on question number 4.



The student's answer is "pendapat Nita benar karena jajargenjang sebuah segiempat yang sisi berlawanannya sama panjang" means "Nita's opinion is correct because a parallelogram is a quadrilateral whose opposite sides are the same length."

Figure 6.
Example of student's answer on question number 4.



The student's answer is "Susi yang benar, jajargenjang tidak mempunyai 4 sisi" means "Susi is correct, a parallelogram does not have 4 sides"

The response to the fourth problem in Figures 5 and 6 indicates that students cannot recognize the relationship between the properties of a geometric figure and those of different shapes using informal deduction.

The questions for geometric thinking level 0 are questions number 1 and 2, with the following indicators: 1) recognizing a quadrilateral according to its overall shape; 2) determining and grouping quadrilaterals according to their shape; 3) drawing a rectangle; 4) identifying the rectangle from the image found; and 5) giving examples of objects that have the same shape as the rectangles encountered. Based on the test results, only one-third of the students were able to do this question correctly and qualify the existing indicators, so only one-third of the total students were at level 0 (visualization). The question for geometric thinking level 1 is question number 3, with the following indicators: 1) draw and identify a quadrilateral according to its characteristics; 2) identify the characteristics of a quadrilateral; 3) describe the class of a shape based on its properties; and identify the rectangle according to the picture and its characteristics. Based on the test results, only a few students could do this question correctly and qualify the existing indicators. The question for geometric thinking level 2 is question number 4, with the following indicators: 1) making implications; 2) identifying the minimal properties to draw a shape; 3) creating and using definitions; and 4) providing more than one explanation or approach.

Based on the test results, all subjects have not been able to solve the problem correctly and have yet to qualify the existing indicators. It means all subjects still need to be at level 2 (informal deduction) of geometric thinking. The level of thinking of these students is not appropriate to Piaget's theory of development and the opinion of the expert of geometry van Hiele, namely Hoffer and Crowley, that students at the high school stage are at stage 0 (visualization) to stage 2 (informal deduction). Only a few junior high school students in Kambera District, East Sumba Regency, East Nusa Tenggara reached level 1 (analysis). This is in line with Burger & Shaughnessy's (1986) research, which states that the highest level of geometric thinking of junior high school students is level 2 (informal deduction). However, it turns out that only a small number of junior high school students in Kambera have a geometric thinking level of level 0 (visualization). This means that many students still have not reached the geometric thinking level of level 0. This aligns with (Cesaria, 2021; Abdullah, 2013; Chong, 2001), who state that most junior high school students are still at stages 1 (visualization) and 2 (analysis) of the geometric thinking level. Of course, this is different from Van de Wall's statement that junior high school students are expected to have reached a level of abstraction in geometric thinking (Muhassanah, 2014).

Many factors affect the level of students' geometric thinking ability, such as 1) lack of adequate learning experience, namely the use of teaching aids, or 2) constructivism learning that can lead students to build understanding and reflect a concept correctly. Teachers do not provide sufficient opportunities for students to build their own knowledge, which is one reason students do not master mathematics. Most students learn mathematics directly in a finished form (formal) because mathematics is seen as a procedural and mechanical process (Andang, 2018). It is appropriate to van Hiele's geometric theory, which states that the speed/success in moving from one thinking stage to the next is more influenced by the content and learning methods experienced by students (Crowley: 1987). It is the teacher's responsibility to organize learning and provide learning experiences that are suitable for students' developmental stage, as well as to develop students' geometric thinking levels. Referring to the indicators of geometric thinking ability used in this study, teachers need to strengthen students' understanding at the visualization and analysis levels so that they can achieve a higher level of geometric thinking. According to (Gutierrez A, 1991), students recognize shapes and geometric configurations at the visualization level based on their overall appearance. At this stage, students identify quadrilaterals by observing their complete form, naming or labeling them with standard or nonstandard terminology. They can construct, draw, or replicate a quadrilateral and verbally describe its appearance. Students also compare and categorize quadrilaterals based on their visual characteristics. However, their sorting often relies on imprecise visual details and irrelevant attributes while overlooking essential properties. They do not analyze the components or properties of quadrilaterals to identify or name them accurately and cannot create formal definitions for different types of quadrilaterals. Instead, their definitions are limited to describing the physical features of the shapes. Students examine geometric figures at the analysis level by focusing on their components and the relationships between them. They empirically determine the properties of a class of figures and use those properties to solve problems. At this stage, students identify the individual components of quadrilaterals and use appropriate terminology to describe these components and their relationships. They compare shapes by analyzing the relationships among their components and sorting quadrilaterals based on specific properties, distinguishing instances of a class from non-instances.

Students interpret and apply verbal descriptions of a figure's properties to draw or construct it. They discover and generalize the properties of specific quadrilaterals through empirical observation. Additionally, they describe a class of figures using its defining properties and recognize how the properties of one class may apply to another, comparing different classes of figures based on their attributes. However, students at this level cannot logically relate properties to one another, classify quadrilaterals systematically, or explain hierarchical subclass relationships.

Several studies have addressed the low level of students' geometric thinking. (Siew, 2013) applied van Hiele phase-based learning using tangrams to determine its effect on students' geometric thinking levels (visualization level and analysis level). The findings of his study revealed that using Van Hiele's learning phases with tangrams significantly enhanced geometric thinking at the first (visual) and second (analysis) levels among students of high, moderate, and low ability. The most significant improvement was observed in low-ability students compared to their moderate and high-ability peers. Therefore, Van Hiele's phase-based learning with tangrams can be effectively used in primary school mathematics to help students reach higher levels of geometric thinking. This aligns with Siew's findings (Abdullah, 2013), suggesting that applying Van Hiele's Phase-based Learning can improve students' geometric thinking.

At the age of 11/13 years, students' abilities begin to develop from concrete thinking to abstract thinking. Students start to see abstract relationships of geometric objects and develop informal deductions. Therefore, teachers must pay attention to students' geometry learning because it depends on their level of thinking (Clement, 2000; Van Hiele, 1999).

4. Conclusion

Based on the study's results and discussion, it can be concluded that most junior high school students in Kambera District, East Sumba Regency, East Nusa Tenggara are at level 0 of geometric thinking (visualization), with only a tiny number reaching level 1 (analysis). None of the students reached level 2 (informal deduction). The results showed that only one-third of the students are at level 0 (visualization), i.e., students knew geometric pictures by observing the images or models. At this level, half of the students still need to improve in classifying the types of quadrilaterals. At level 0, there are still students who are wrong in identifying examples of quadrilaterals based on their overall appearance and students who do not consider the components or properties of quadrilaterals to identify or name quadrilaterals. Some students still have not been able to name or label quadrilaterals and other geometric configurations and use standard or non-standard names correctly. At level 1, only a few students gave answers; the rest did not. The answers given are correct. It means that some students have been able to identify the components of quadrilaterals, interpret and use verbal descriptions of a shape based on its properties and use these properties to draw or create the shape, and can find the properties of certain quadrilaterals empirically and generalize the properties for the class of quadrilaterals. While at level 2, none of the students gave answers related to the problems given. The students' geometric thinking levels are influenced by the limited learning experiences and the lack of constructivist teaching approaches that could help students develop a proper understanding and reflect on concepts correctly. Therefore, teachers need to teach geometry in a way that aligns with the students' cognitive levels.

5. Aknowledgments

Universitas Kristen Wira Wacana Sumba supported this study. We also want to thank the teachers and students who participated in it.

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