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Application of Dynamic Programming in Determining The Shortest Route PT JNE Using Backward Recursive Equation

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Article Info	Abstract
Article history: Received : June 18. 2024 Revised : July 29, 2024 Accepted : July 30, 2024 Available online : July 31, 2024 https://doi.org/10.33541/edumatsains.v9i1.5982	Technological progress speeds up human movement, the distribution of goods, and service provision. PT JNE, a prominent firm in Indonesia, concentrates on delivering goods swiftly and with comprehensive services.. On the other hand, problems include daily changes in delivery, delays, less than optimal routes, and expensive gasoline. The shortest ideal route for the transportation of products is found using dynamic programming and a backward recursive equation technique in this study. The Medan Belawan District as well as the JNE Medan Representative Office provided the data that was used. The findings show that a route of a → f → d → c → g → b → e → h → a is the ideal total distance for goods delivery using a dynamic programming graph is 12,2 km, with an average courier covering 16,5 km. Based on data analysis, this shows a 26% increase in efficiency over the current routes. The layout of the routes makes it easier for couriers to choose the fastest route. **Keywords:** shortest route, dynamic programming, backward recursive equations

1. Introduction

The rapid technological progress in this era requires speeding up in multiple areas, such as human mobility, goods delivery, and service provision. Online systems have emerged as solutions that streamline tasks like document transmission and commercial transactions. Additionally, this system facilitates remote work, increasing access to things, thereby revolutionizing various aspects of daily life and business operations.

Expedition or delivery services function as middlemen between buyers and sellers, arranging for the timely delivery of products. There are numerous rival expedition businesses operating now, each with unique tactics (Sianturi & Puspasari, 2023). Delivery services are essential for many different needs and sectors, but they are particularly important for online purchases and sales. This is what the smoothness of these transactions has contributed to a substantial change in the dynamics of online commerce (Esanata, 2019).

The expedition involves moving different kinds of goods from one place to another with the goal of improving transportation and delivery efficiency (Nurmala & Thamrin, 2023). Selecting the right shipping routes is vital for enhancing customer service quality (Rinaldi et al., 2021). One of the top delivery services in Indonesia is JNE, which specializes in expedited transportation of goods across provinces and regions. Using an integrated online communication system, JNE has expanded its service network throughout Indonesia to give customers access to the most recent information (JNE, 2018).

A salesman delivering goods must meticulously plan the order of stops from the starting point to the destination, making only one visit per location in a single trip, and carefully consider travel time and distance in order to maximize efficiency (Hazizah et al., 2023). Distribution, often known as delivery, is the process of moving products and services from the seller to the final customer via distribution networks, according to (Sirait & Lubis, 2023). The process of moving products or services from the producer to the customer is known as distribution or delivery (Hariati et al., 2021). Determining the shortest path that can save time and money during the distribution process is therefore essential (Chandra & Setiawan, 2018).

Dynamic programming is a problem-solving methodology that entails decomposing the solution into successive stages, where resolving a specific problem can be viewed as the outcome of a sequence of interlinked decisions (Agnezia & Winarno, 2022). This technique employs tables to scrutinize and record the outcomes of computations at each stage, thereby facilitating a more detailed understanding of the calculation solutions (Cormen et al., 2022).

Expert system creation requires the use of inference engines (Moore & Quintero, 2019). The two types of inference methods used in rule-based expert systems are the Forward Method and the Backward Method (Alfaris et al., 2022). To find the shortest delivery route, this study uses the Backward Recursive Equation in conjunction with the Dynamic Programming approach. This method is relatively easy to implement, especially in systems with a hierarchical or repetitive structure. Additionally, this method can save time and resources by computing solutions for partial problems and using these results to obtain the overall solution, making it more efficient than searching for explicit solutions to the entire problem. According to (Elsa et al., 2023), this approach may choose the optimal route among multiple options by considering factors like the destination's address and city. The Backward Method, also called the Backward Recursive Equation, is a method that starts with drawing preliminary findings.

To validate the conclusions, the next step is to align hypotheses with the facts existing in the knowledge base (Irawan & Fitrialdy, 2020). As opposed to looking for clear answers to every issue, this approach may be more effective (Sari et al., 2021). Dynamic programming can be used to solve these problems. These problems can be solved using a variety of dynamic programming techniques, each specifically designed to address the unique optimization features of the individual problems (Sagala et al., 2022). The application used is Google Maps, which shows routes as straightforward weighted graphs. A weighted graph, according to (Rahayuningsih, 2018), is one in which each edge is given a non-negative value or weight. This makes the distances between spots easier to see.

The four steps listed below can be used to create a dynamic programming method, according to (Cormen et al., 2022):

1. Describe the ideal solution's structure.
2. Use recursion to define the optimal solution's value.
3. Calculate the ideal solution's value by using a forward or backward calculation.
4. Formulate the optimal solution based on the calculated data.

Because of its intriguing character as an optimization issue, the Travelling Salesman issue (TSP) has attracted a lot of study interest throughout the years (Sinaga & Marpaung, 2023). TSP essentially includes a salesman who has to visit several sites with specified distances between them, make sure each destination is visited exactly once, and then take the shortest route back to the starting point. The following two equations are derived from the optimality principle:

$$f(i, \emptyset) = c_{i,1}; 2 \leq i \leq n \quad \ldots \qquad (base) \tag{1}$$

$$f(i, S) = min_{j \in S}\{c_{i,j} + f(j, S - \{j, S\} - \{j\})\} \ldots \qquad (base) \tag{2}$$

Next, utilize both equations to solve for the shortest route using dynamic programming.

PT Jalur Nugraha Eka Kurir (JNE), a leading courier company in Indonesia, faces daily variations in shipment volume and destinations. Challenges such as delivery delays, inefficient routing, and soaring fuel expenses are prevalent. Consequently, optimizing delivery routes becomes imperative to enhance the company's profitability in an industry where efficient transportation and distribution are pivotal for cost-cutting and competitive edge.

2. Methods

Research method is essentially a scientific approach employed to gather data for specific purposes and utilities. Scientific approach implies that the research process is conducted logically and can be elucidated through human reasoning (Rachman et al., 2024). This research utilizes a quantitative descriptive method, focusing on depicting the facts and characteristics of the research object as well as the relationships among its variables. The type of data employed in this research is quantitative data, consisting of numerical information obtained from both primary and secondary sources. The research was conducted over a span of 5 months from October to February 2024. The research took place at the JNE Medan Representative Office located at Gatot Subroto Road, Taman Komplek Tomang Elok Road No. 3, Km 5.5, Medan Sunggal.

The steps involved in processing the data are as follows:

1. Problem Identification: Conducting preliminary studies to obtain initial information and the issues to be addressed.
2. Literature Review: The problem is examined using dynamic programming with backward recursive equations.
3. Data Collection: Distance between delivery points in the Medan Belawan Subdistrict taken from the JNE Medan Representative Office.
4. Data Processing: Initially creating a network diagram (graph model) with the assistance of Google Maps software to find the shortest route.
5. The results of data processing will determine the shortest route based on dynamic programming calculations utilizing backward recursive equations employing the foundational formula (1) and recurrence relation (2).
6. Drawing conclusions.

3. Result and Discussion

3.1 Data Collection

This study focuses on the shipment routes in the Medan Belawan Subdistrict, which are used to transport items from the JNE Medan Representative Office (Drop Point/DP) to the delivery places that are visited the most frequently. The following table provides details on the delivery points:

Table 1. ***Place Names and Delivery Addresses***

No	Place	Symbol	Address
1	Salam Mosque	a	Harbor Highway, Belawan II, Medan Belawan City
2	Rismanda's Customer	b	Bagan Deli Road, Alley 5 Veteran, Medan Belawan
3	PT Pelindo Multi Terminal	c	Harbor Ring Road I, Belawan II
4	PT Smart TBK Belawan	d	New Balmera road III Belawan, Belawan II, Medan Belawan
5	Belawan Harbor Police Station	e	Harbor highway, Bagan Deli, Medan Belawan City
6	PT Sea Asih Lines	f	Bangka Timur Road, No. 104, Medan Belawan
7	PT Musim Mas Belawan	g	Sulawesi Road II No 1, Ujung Baru, Medan Belawan
8	PT Graha Segara Belawan	h	Gabion Port Highway, Neighborhood 12 Belawan

Table 2. ***Distance Between Places in Kilometers***

	a	b	c	d	e	f	g	h
a	0	3,1	2,1	1,2	3,3	1,1	2,7	4,6
b	3,1	0	2,5	2	0,4	2,4	1,9	1,7
c	2,1	2,5	0	1	2,6	1	1,3	3,9
d	1,2	2	1	0	2,2	0,6	1,6	3,5
e	3,3	0,4	2,6	2,2	0	2,6	2	1,3
f	1,1	2,4	1	0,6	2,6	0	2,1	3,9
g	2,7	1,9	1,3	1,6	2	2,1	0	3,2
h	4,6	1,7	3,9	3,5	1,3	3,9	3,2	0

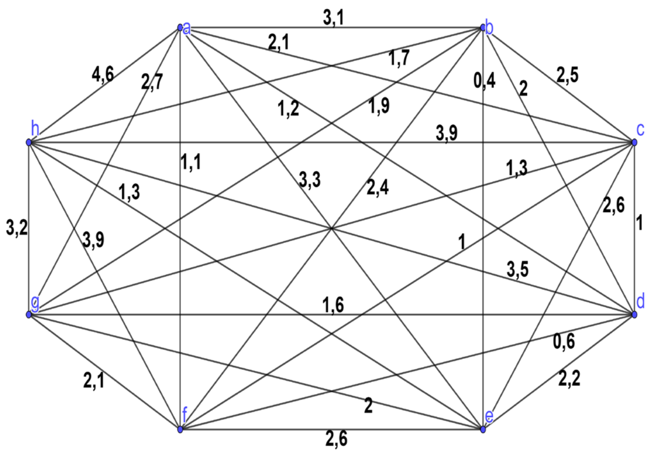

Figure 1. ***Distance Graph and Courier Travel Routes***

3.2 Calculating Shortest Routes with Dynamic Programming

In the calculation of shortest routes using dynamic programming, the process operates in a backward manner (backward recursive equations), with the optimal result determined from stage n and ending at stage 1.

In the first stage of the calculation, the following base equation is used:

$$f(i,a) = c_{i,1}$$

Where, i : represents the name of the location or transportation point (stage),

$f(i,a)$: denotes the optimal value function,

$c_{i,j}$: represents the output value at stage

Thus, the results are obtained as follows:

$f(b,a) = 3{,}1$ $\quad f(e,a) = 3{,}3$ $\quad f(h,a) = 4{,}6$

$f(c,a) = 2{,}1$ $\quad f(f,a) = 1{,}1$

$f(d,a) = 1{,}2$ $\quad f(g,a) = 2{,}7$

In the first stage, this yields 7 alternative routes that can be taken, with a representing their status. Then, in the second stage and onwards, the calculation of the shortest routes utilizes the following recurrence equation:

$$F(i,S) = \min_{j \in s}\{c_{i,j} + f(j, S - \{j\})\}\, untuk\ |S| = 1$$

where, S : represents the states or possible conditions that may occur at each stage of the dynamic programming problem.

The following results are obtained:

$f(b,c) = \min C_{bc} + f(c,a) = 2{,}5 + 2{,}1 = 4{,}6$

$f(b,d) = \min C_{bd} + f(d,a) = 2 + 1{,}2 = 3{,}2$

$f(b,e) = \min C_{be} + f(e,a) = 0{,}4 + 3{,}3 = 3{,}7$

$f(b,f) = \min C_{be} + f(f,a) = 2{,}4 + 1{,}1 = 3{,}5$

$f(b,g) = \min C_{bg} + f(g,a) = 1{,}9 + 2{,}7 = 4{,}6$

$f(b,h) = \min C_{bh} + f(h,a) = 1{,}7 + 4{,}6 = 6{,}3$

$f(c,b) = \min C_{cb} + f(b,a) = 2{,}5 + 3{,}1 = 5{,}6$

$f(c,d) = \min C_{cd} + f(d,a) = 1 + 1{,}2 = 2{,}2$

$f(c,e) = \min C_{ce} + f(e,a) = 2{,}6 + 3{,}3 = 5{,}9$

$f(c,f) = \min C_{cf} + f(f,a) = 1 + 1{,}1 = 2{,}1$

$f(c,g) = \min C_{cg} + f(g,a) = 1{,}3 + 2{,}7 = 4$

$f(c,h) = \min C_{ch} + f(h,a) = 3{,}9 + 4{,}6 = 8{,}5$

$f(d,b) = \min C_{db} + f(b,a) = 2 + 3{,}1 = 5{,}1$

$f(d,c) = \min C_{dc} + f(c,a) = 1 + 2{,}1 = 3{,}1$

$f(d,e) = \min C_{de} + f(e,a) = 2{,}2 + 3{,}3 = 5{,}5$
$f(d,f) = \min C_{df} + f(f,a) = 0{,}6 + 1{,}1 = 1{,}7$
$f(d,g) = \min C_{dg} + f(g,a) = 1{,}6 + 2{,}7 = 4{,}3$
$f(d,h) = \min C_{dh} + f(h,a) = 3{,}5 + 4{,}6 = 8{,}1$
$f(e,b) = \min C_{eb} + f(b,a) = 0{,}4 + 3{,}1 = 3{,}6$
$f(e,c) = \min C_{ec} + f(c,a) = 2{,}6 + 2{,}1 = 4{,}7$
$f(e,d) = \min C_{ed} + f(d,a) = 2{,}2 + 1{,}2 = 3{,}4$
$f(e,f) = \min C_{ef} + f(f,a) = 2{,}6 + 1{,}1 = 3{,}7$
$f(e,g) = \min C_{eg} + f(g,a) = 2 + 2{,}7 = 4{,}7$
$f(e,h) = \min C_{eh} + f(h,a) = 1{,}3 + 4{,}6 = 5{,}9$
$f(f,b) = \min C_{fb} + f(b,a) = 2{,}4 + 3{,}1 = 5{,}6$
$f(f,c) = \min C_{fc} + f(c,a) = 1 + 2{,}1 = 3{,}1$
$f(f,d) = \min C_{fd} + f(d,a) = 0{,}6 + 1{,}2 = 1{,}8$
$f(f,e) = \min C_{fe} + f(e,a) = 2{,}6 + 3{,}3 = 5{,}9$
$f(f,g) = \min C_{fg} + f(g,a) = 2{,}1 + 2{,}7 = 4{,}8$
$f(f,h) = \min C_{fh} + f(h,a) = 3{,}9 + 4{,}6 = 8{,}5$
$f(g,b) = \min C_{gb} + f(b,a) = 1{,}9 + 3{,}1 = 5$
$f(g,c) = \min C_{gc} + f(c,a) = 1{,}3 + 2{,}1 = 3{,}5$
$f(g,d) = \min C_{gd} + f(d,a) = 1{,}6 + 1{,}2 = 2{,}8$
$f(g,e) = \min C_{ge} + f(e,a) = 2 + 3{,}3 = 5{,}3$
$f(g,f) = \min C_{gf} + f(f,a) = 2{,}1 + 1{,}1 = 3{,}2$
$f(g,h) = \min C_{gh} + f(h,a) = 3{,}2 + 4{,}6 = 7{,}8$
$f(h,b) = \min C_{hb} + f(b,a) = 1{,}7 + 3{,}1 = 4{,}8$
$f(h,c) = \min C_{hc} + f(c,a) = 3{,}9 + 2{,}1 = 6$
$f(h,d) = \min C_{hd} + f(d,a) = 3{,}5 + 1{,}2 = 4{,}7$
$f(h,e) = \min C_{he} + f(e,a) = 1{,}3 + 3{,}3 = 4{,}6$
$f(h,f) = \min C_{hf} + f(f,a) = 3{,}9 + 1{,}1 = 5$
$f(h,g) = \min C_{hg} + f(g,a) = 3{,}2 + 2{,}7 = 5{,}9$

In the second stage, 42 alternative routes were generated, each having (b, c, d, e, f, g, h) as their statuses. This procedure persisted until the eighth stage, revealing that the optimal total distance for delivering goods at PT Jalur Nugraha Eka Kurir (JNE) Medan is 12,2 km, with the obtained route being of a → f → d → c → g → b → e → h → a.

3.3 Comparison Analysis of Courier Travel Data with Data Processed Using Dynamic Programming

3.3.1 Common Courier Travel Distance

The following is the route commonly taken by couriers in the process of delivering goods to customer locations:
a → f → c → e → b → d → g → h → a.
With a total travel distance of 16,5 km.

3.3.2 Processed Data Distance

The following is the route obtained from data processing using dynamic programming:
a → f → d → c → g → b → e → h → a.
With a total travel distance of 12,2 km.

Thus, the efficiency of distance between the existing route and the route resulting from data processing using dynamic programming is:

$$\%\ distance\ savings = \frac{old\ distance - new\ distance}{old\ distance}\ x\ 100\%$$

$$\%\ distance\ savings = \frac{16{,}5 - 12{,}2}{16{,}5}\ x\ 100\%$$

$$= 26{,}06\%$$

$$= 26\%\ \text{(rounded)}$$

4. Conclusion

Based on the analysis and discussion conducted, it may be determined that the best routes for couriers to carry items to clients can be found by using the dynamic programming approach with graph applied backward recursive equations. This is demonstrated by the use of graphs in problem-solving procedures, where the shortest path between each node on pathways with minimal distance is determined. The findings reveal that the optimal total distance for goods delivery at PT Jalur Nugraha Eka Kurir (JNE) Medan is 12,2 km, following the route a → f → d → c → g → b → e → h → a, whereas couriers typically travel a distance of 16,5 km. This showcases an efficiency improvement of 26% when comparing existing routes with those derived from data analysis. This study is supported by previous relevant research, such as the work conducted by (Karthikeyan et al., 2014) and (Elsa et al., 2023), which discusses the application of dynamic programming in shortest path problems.

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